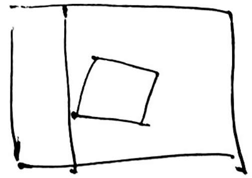
经 ~ ①



$$\frac{0}{0}=1$$

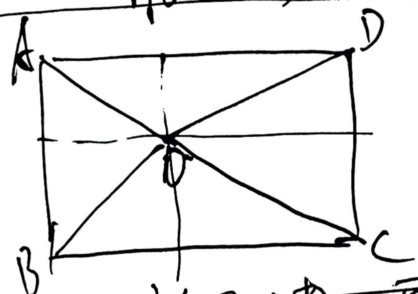
$(\Rightarrow)\ 0=0\times 1$

② 向量法

③ 特殊化策略

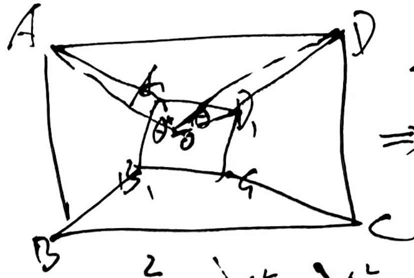
{ 同手法
同结果

缩为一个点



$OA^2+OC^2=OB^2+OD^2$

该结论能否解决一般情况？



$\triangle OAD \sim \triangle OA_1D_1$

$\Rightarrow \triangle OAA_1 \sim \triangle ODD_1$

极端化

$AA_1^2 = OA^2+OA_1^2-2OA\cdot OA_1\cos\theta$

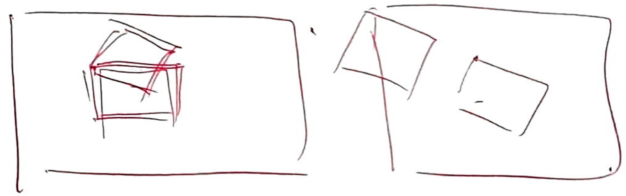
$BB_1^2 = OB^2+OB_1^2-2OB\cdot OB_1\cos\theta$

$CC_1^2 = OC^2+OC_1^2-2OC\cdot OC_1\cos\theta$

$DD_1^2 = OD^2+OD_1^2-2OD\cdot OD_1\cos\theta$

得

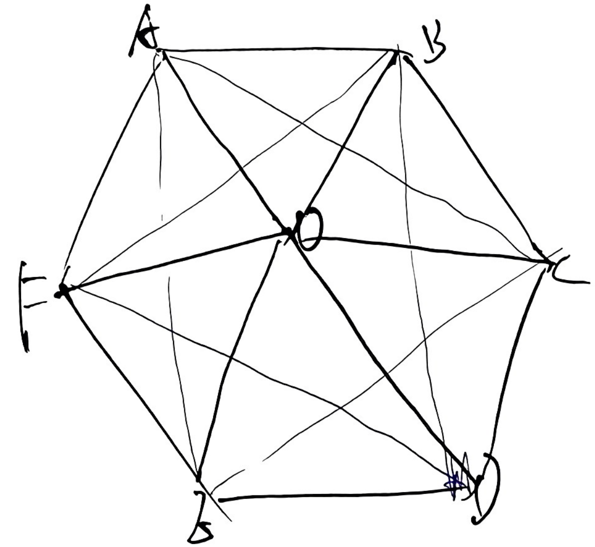
一般化

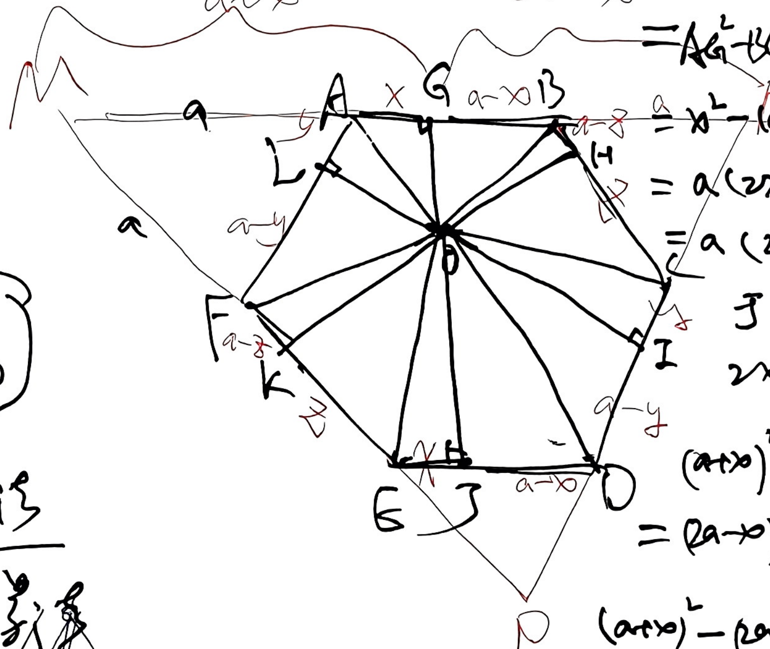


小范围缩为一个点

$$OA^2+OC^2+OE^2=OB^2+OD^2+OF^2$$

$$OA^2+OD^2=OB^2+OE^2=OC^2+OF^2$$?



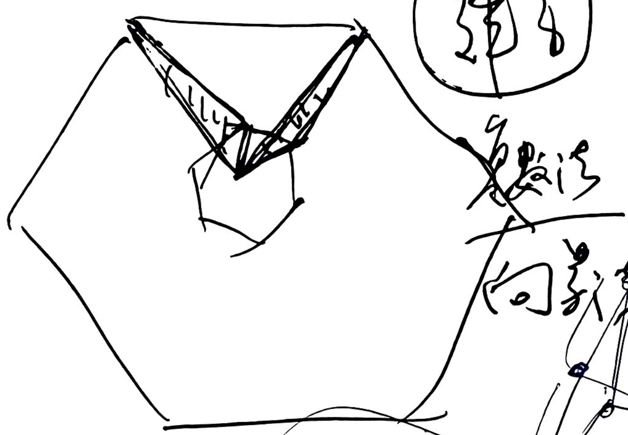


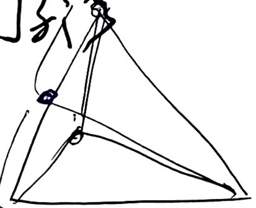
$$OA^2-OB^2+OC^2-OD^2+OE^2-OF^2$$
$$=AG^2-BG^2+CI^2-DI^2+EK^2-FK^2$$
$$=x^2-(a-x)^2+\ \cdots+\ \cdots$$
$$=a(2x-a)+\cdots+\cdots$$
$$=a(2x+2y+2z-3a)$$

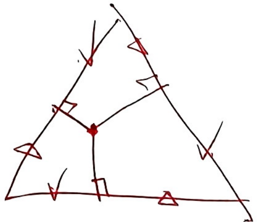
于是只需证 $2x+2y+2z=3a$?

$$(a+x)^2+(a+y)^2+(a+z)^2$$
$$=(2a-x)^2+(2a-y)^2+(2a-z)^2$$

$$(a+x)^2-(2a-x)^2+(a+y)^2-(\ )+\cdots=0$$
$$3a(2x-a)+\cdots+\cdots=0$$
$$3a(2x-a+2y-a+2z-a)=0$$
$$\therefore 2x+2y+2z=3a$$







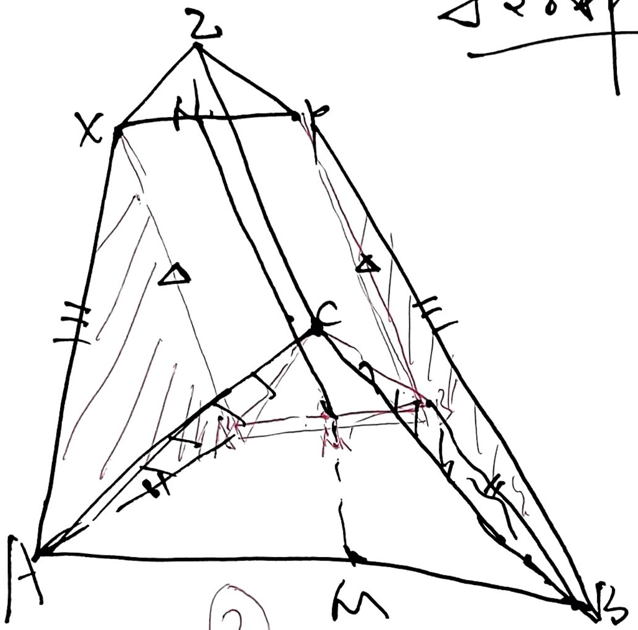
辅

多变问

向量

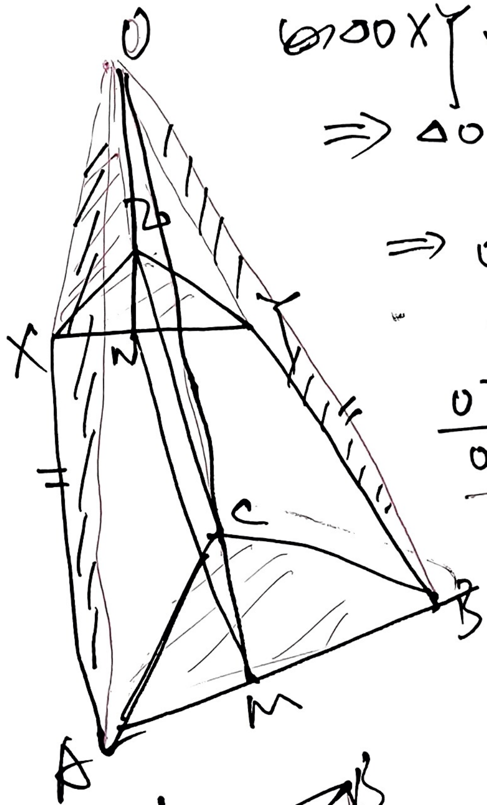
如意算盘

特殊化



作辅助的中心

光速解法



由△OXY∽△OAB

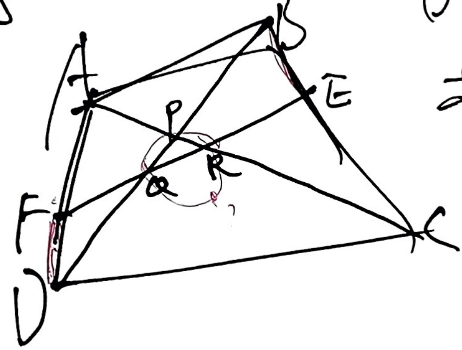
⇒ △OXA∽△OYB

⇒ OA=OB

OX=OY

$\frac{OZ}{OC}=\frac{XZ}{AC}=\frac{ZN}{CM}$

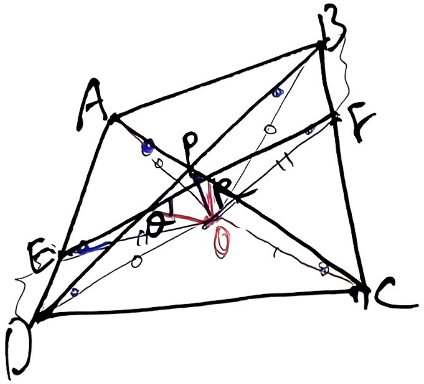
45 46



BC=AD, BE=DF

求证△PQR过定点

(可能是)



$\angle OPR = \angle OPC = \angle OBC$

$\Uparrow$

P. O. C. B 共圆

$\angle OQR = \angle EDO = \angle ODA$

$\Uparrow$

Q. E. D. O 共圆

AED 与 CFB 相似

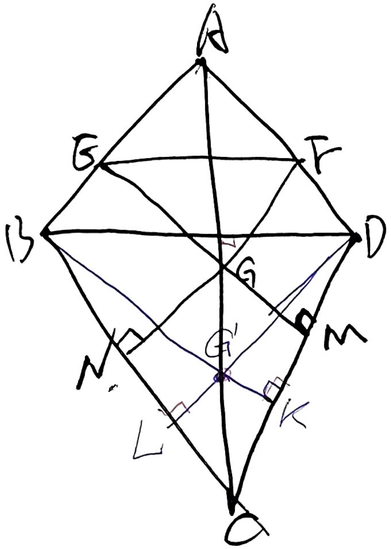
~~△OAD ~ △OCB~~

△DOA 与 △BOC

△AOC. △BOD △EOF

为相似的三角形.

好.



作BK⊥CD于K交AC于G'，连DG'并延长交BC于L.

$BK \parallel EM \Rightarrow \dfrac{AE}{EB} = \dfrac{AG}{GG'}$

$EF \parallel BD \Rightarrow \dfrac{AE}{GB} = \dfrac{AF}{FD}$

$\Rightarrow \dfrac{AG}{GG'} = \dfrac{AF}{FD}$

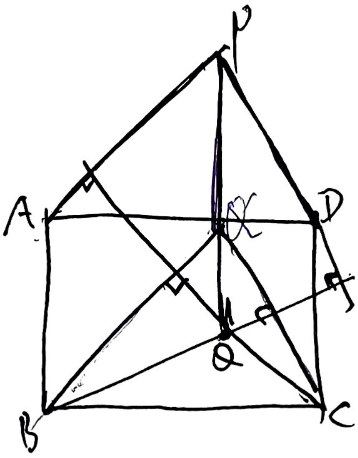
$\Rightarrow FG \parallel DG'$

即 $FN \parallel DL$，$FN \perp BC$ $\Rightarrow DL \perp BC$

∴ G'为△BCD重心

∴ $AC \perp BD$

5



∵作PQ'⊥AB }
AB⊥BC } ⇒ PQ'⊥BC

Q为△Q'BC之垂心

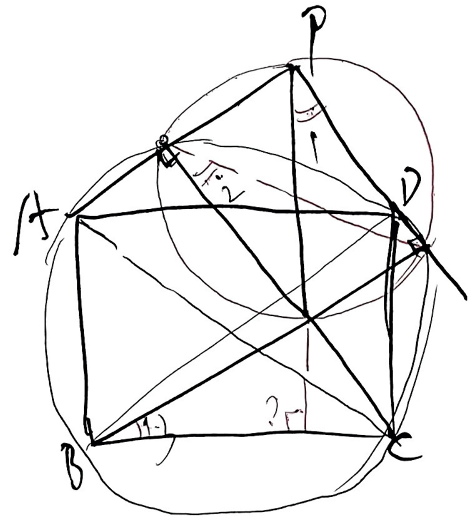
∴QQ'⊥BC

过Q' ~~有且只有一~~ 条垂线

与BC垂直

∴P、Q'、Q 三点共线

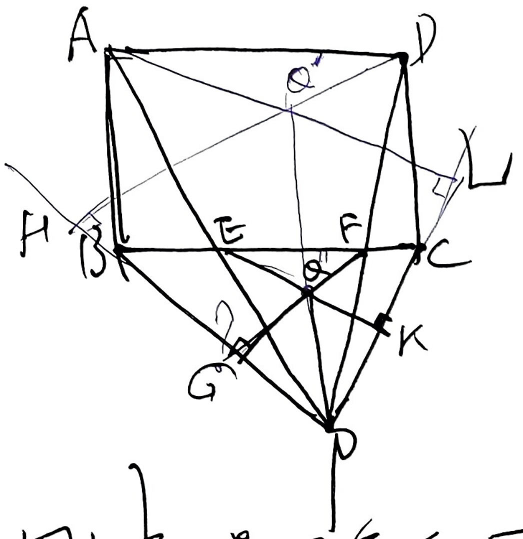
∴PQ⊥BC



∠1=∠2=∠3

48

49



刚才的结论能利用吗:

$PQ' \perp BC$?

下证 P, Q, Q' 共线

变通?

$FG \parallel DH \Longleftrightarrow \dfrac{PQ''}{Q'Q''} = \dfrac{PG}{HG}$ (?)

$Q''K \parallel Q'L \Longleftrightarrow \dfrac{PQ''}{Q'Q''} = \dfrac{PK}{KL}$

只需证 $\dfrac{PG}{HG} = \dfrac{PK}{KL}$

$FG \parallel DH \Rightarrow \dfrac{PG}{HG} = \dfrac{PF}{DF}$

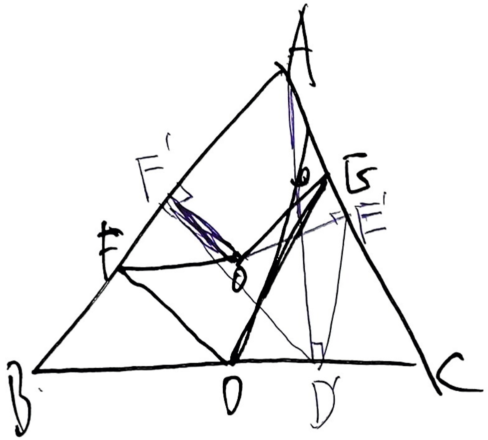
$EK \parallel AL \Rightarrow \dfrac{PK}{KL} = \dfrac{PE}{AE}$

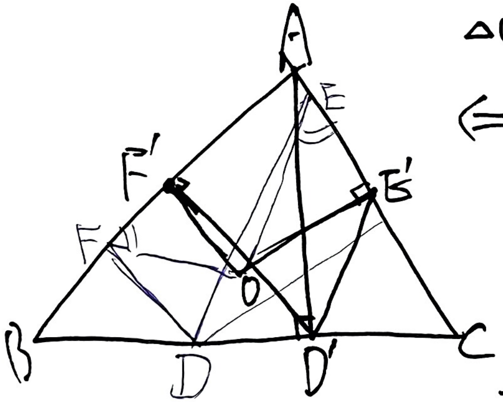
只需证 $\dfrac{PF}{DF} = \dfrac{PE}{AE}$

$\Uparrow$

$EF \parallel AD$

例3





$\triangle OE'E \backsim \triangle OF'F$

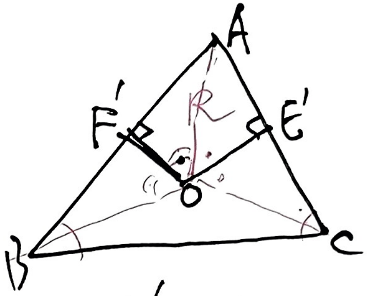
$$\Leftarrow \frac{EE'}{FF'} = \frac{OE'}{OF'}$$

$$\frac{EE'}{FF'} = \frac{EE'}{DD'} \cdot \frac{DD'}{FF'}$$

$$= \frac{OE'}{OD'} \cdot \frac{BD'}{BF'}$$

$$= \frac{\frac{1}{2}AC}{AC\cos C} \cdot \frac{AB\cos B}{\frac{1}{2}AB}$$

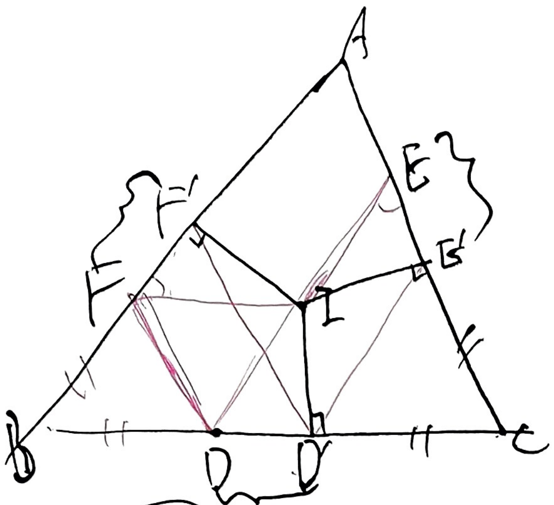
$$= \frac{\cos B}{\cos C}$$



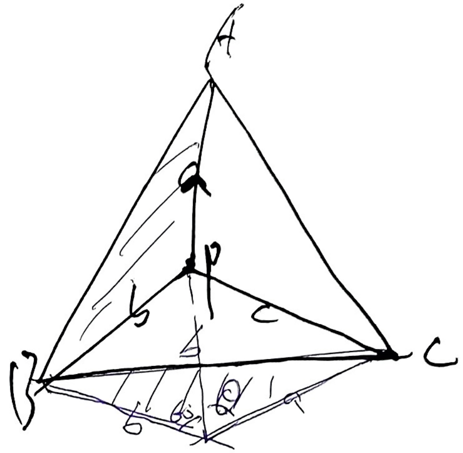
$$\frac{OE'}{OF'} = \frac{\cancel{OA}\cdot\cos\angle AOE'}{\cancel{OA}\cos\angle AOF'}$$

$$= \frac{\cos B}{\cos C}$$

没说理 ?



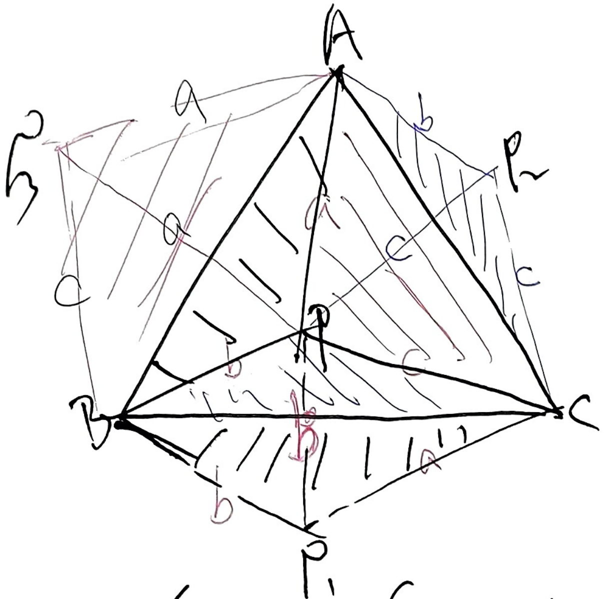
$$\frac{EE'}{FF'} = \frac{IE'}{IF'} = 1$$

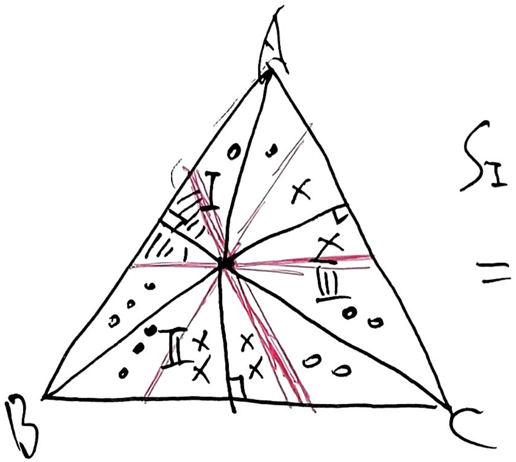


$$\frac{\sqrt{3}}{4} BC^2 = \frac{\sqrt{3}}{4}\left(a^2+b^2-2ab\cos\underbrace{(60+\theta)}_{\left(\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta\right)}\right)$$

$$\cos\theta = \frac{a^2+b^2-c^2}{2ab} \qquad \sin\theta = ?$$

15





$S_{\mathrm{I}} + S_{\mathrm{II}} + S_{\mathrm{III}}$

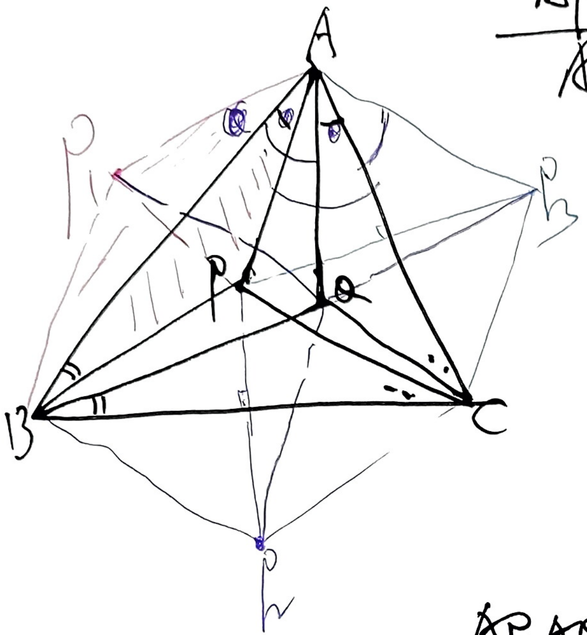
$= \frac{1}{2} S_{\triangle ABC}$

$2S_{\triangle ABC} = S_{六边形}$

$= \frac{\sqrt{3}}{4}(a^2 + b^2 + c^2) + 3\sqrt{p(p-a)(p-b)(p-c)}$

$S_{\triangle ABC} = \frac{\sqrt{3}}{8}(a^2+b^2+c^2) + \frac{3}{2}\sqrt{p(p-a)(p-b)(p-c)}$

(其中 $p = \frac{a+b+c}{2}$)



$$\frac{AP\cdot AQ}{AB\cdot AC}+\frac{BP\cdot BQ}{AB\cdot BC}+\frac{CP\cdot CQ}{BC\cdot AC}=1$$

$$S_{六边形}=\underline{2S_{\triangle ABC}}$$

$$=S_{\triangle QAP_1}+S_{\triangle QAP_3}+\cdots$$

$$=\frac{1}{2}\cdot AP_1\cdot AQ\sin\angle P_1AQ+\frac{1}{2}AQ\cdot AP_3\sin\angle P_3AQ+\cdots$$

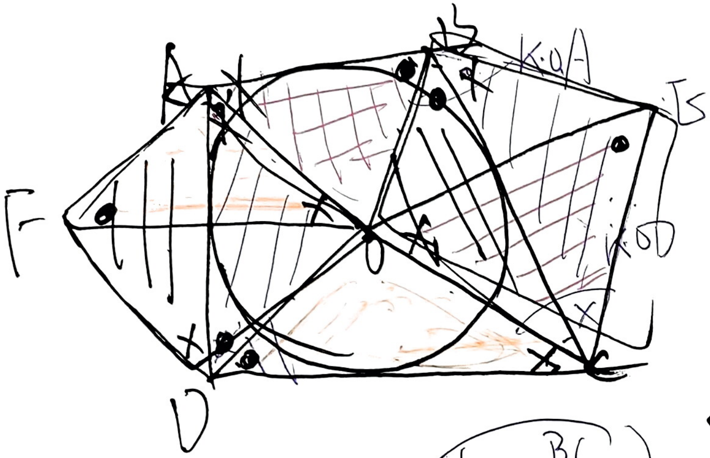
$$=\frac{1}{2}AP\cdot AQ\sin A+\frac{1}{2}AQ\cdot AP\sin A+\cdots$$

$$=AP\cdot AQ\sin A+BP\cdot BQ\sin B+CP\cdot CQ\sin C$$

$$1=\frac{AP\cdot AQ\sin A}{2S_{\triangle ABC}}+\frac{BP\cdot BQ\sin B}{2S_{\triangle ABC}}+\frac{CP\cdot CQ\sin C}{2S_{\triangle ABC}}$$

$$=\frac{AP\cdot AQ\cancel{\sin A}}{AB\cdot AC\cancel{\sin A}}+\frac{BP\cdot BQ\cancel{\sin B}}{AB\cdot BC\cancel{\sin B}}+\frac{CP\cdot CQ\cancel{\sin C}}{BC\cdot AC\cancel{\sin C}}$$

$$OA\cdot OC+OB\cdot OD=\sqrt{abcd}$$



$$k=\frac{BC}{AD}$$

$$OB\cdot CE+OC\cdot BE=BC\cdot OE$$

$$OB\cdot\frac{\cancel{BC}}{AD}\cdot OD+OC\cdot\frac{\cancel{BC}}{AD}\cdot OA=\cancel{BC}\cdot OE$$

$$OB\cdot OD+OC\cdot OA=AD\cdot OE \quad ①$$

$$OB\cdot OD+OC\cdot OA=BC\cdot OF \quad ②$$

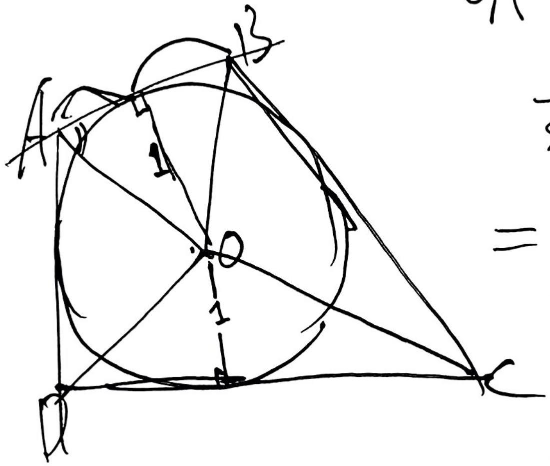
①×②

$$\Rightarrow OA\cdot OC+OB\cdot OD=\sqrt{AD\cdot BC\cdot OE\cdot OF}$$

只需证 $OE\cdot OF=AB\cdot CD$

$$\frac{AB}{OE}=\frac{OA}{OC}\qquad \frac{CD}{OF}=\frac{OC}{OA}$$

$$\frac{AB}{OE}\cdot\frac{CD}{OF}=1 \quad \therefore OE\cdot OF=AB\cdot CD$$



$$OA\cdot OC + OB\cdot OD = \sqrt{abcd}$$

$$\frac{1}{\sin\frac{A}{2}}\cdot\frac{1}{\sin\frac{C}{2}} + \frac{1}{\sin\frac{B}{2}}\cdot\frac{1}{\sin\frac{D}{2}}$$

$$= \sqrt{\left(\cot\frac{A}{2}+\cot\frac{B}{2}\right)\left(\cot\frac{B}{2}+\cot\frac{C}{2}\right)\left(\cot\frac{C}{2}+\cot\frac{D}{2}\right)\left(\cot\frac{D}{2}+\cot\frac{A}{2}\right)}$$

$$\Longleftrightarrow \frac{\sin\frac{A}{2}\sin\frac{C}{2}+\sin\frac{B}{2}\sin\frac{D}{2}}{\cancel{\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\sin\frac{D}{2}}} = \frac{\sqrt{\sin\frac{A+B}{2}\sin\frac{B+C}{2}\sin\frac{C+D}{2}\sin\frac{D+A}{2}}}{\cancel{\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\sin\frac{D}{2}}}$$

?

$$\cot\frac{A}{2}+\cot\frac{B}{2} = \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}}+\frac{\cos\frac{B}{2}}{\sin\frac{B}{2}} = \frac{\sin\frac{A+B}{2}}{\sin\frac{A}{2}\sin\frac{B}{2}}$$

$$\boxed{\begin{aligned}&A+B+C+D\\&=360°\\&\frac{A+B+C+D}{2}\\&=180°\end{aligned}}$$

← 隐含条件

$$\Longleftrightarrow \sin\frac{A}{2}\sin\frac{C}{2}+\sin\frac{B}{2}\sin\frac{D}{2} = \sin\frac{A+B}{2}\sin\frac{B+C}{2}$$

?

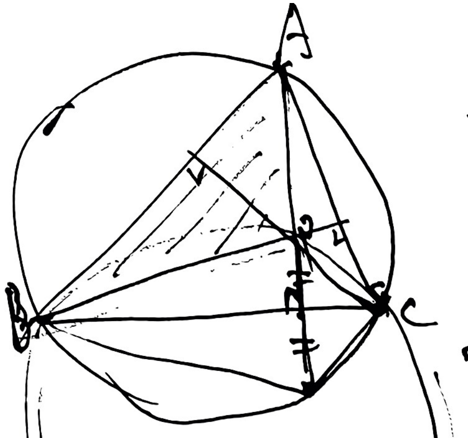
$$\frac{1}{2}\left(\cos\cancel{\frac{A+C}{2}}-\cos\frac{A-C}{2}\right)+\frac{1}{2}\left(\cos\frac{B-D}{2}-\cos\frac{B+D}{2}\right)$$

$$=\frac{1}{2}\left(\cos\cancel{\frac{A+C}{2}}-\cos\frac{A+B+B+C}{2}\right)$$

$360°-\frac{B-D}{2}$

$$\cancel{\left(180°+\frac{B-D}{2}\right)}$$

故



$$S_{\triangle ABC} = S_{\triangle ABP} + S_{\triangle ACP} + S_{\triangle BCP}$$

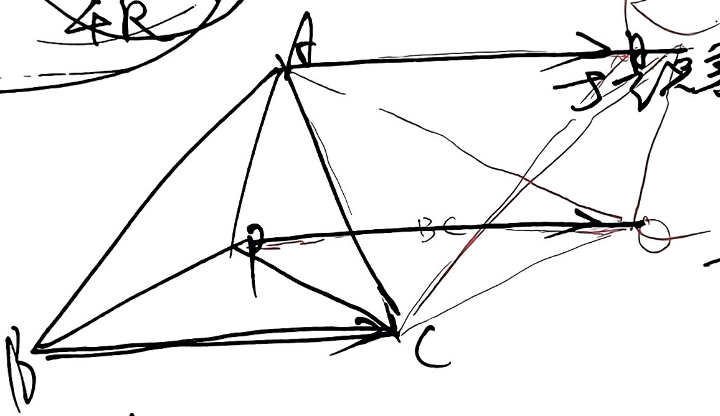
$$\frac{AB\cdot BC\cdot CA}{\cancel{4R}} = \frac{AP\cdot BP\cdot AB}{\cancel{4R}} + \_\_\_ + \_\_\_$$

$$AB\cdot \underbrace{BC\cdot CA}_{PQ} \le AB\,(AP\cdot CQ + PC\cdot AQ)$$

$$= AB\cdot AP\cdot CQ + AB\cdot PC\cdot AQ$$

$$= AB\cdot AP\cdot BP + AB\cdot PC\cdot AQ$$

$$S_{\triangle} = \frac{abc}{\cancel{4R}}$$



手段度意记

$$AB\cdot \cancel{PC}\cdot AQ \le BP\cdot \cancel{CP}\cdot BC + AP\cdot \cancel{CP}\cdot AC$$

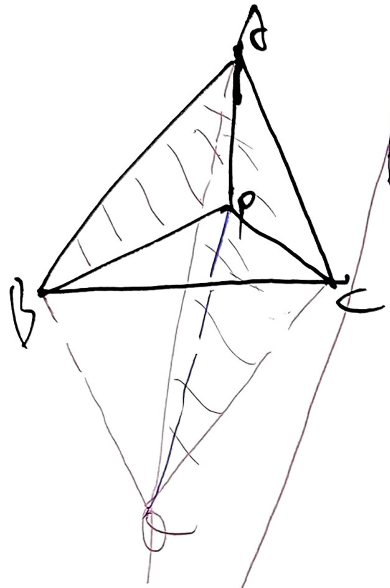
$$\Longleftrightarrow AB\cdot AQ \le BP\cdot BC + AP\cdot AC$$

即 $= AR\cdot AQ \le CQ\cdot AR + RQ\cdot AC$

$$AB\cdot BC\cdot CA \le \cdots + \cdots + \cdots$$

托勒密定理

15



$\triangle ABP \backsim \triangle AQC \Longrightarrow \dfrac{AB}{AQ}=\dfrac{AP}{AC} \Longrightarrow AB\cdot AC = AP\cdot AQ$

$\Longrightarrow \triangle ABQ \backsim \triangle APC \Longrightarrow \dfrac{AB}{AP}=\dfrac{BQ}{PC} \Longrightarrow \boxed{AB\cdot CP = AP\cdot BQ}$

$AB\cdot AC\cdot BC \leq AP\cdot AQ\cdot BC \leq AP(AP+PQ)\cdot BC$

$= AP^2\cdot BC + AP\cdot PQ\cdot BC$

于是只需 $AP\cdot PQ\cdot BC \leq BP^2\cdot AC + CP^2\cdot AB$

$\Longleftrightarrow \cancel{AP}\cdot PQ\cdot BC \leq BP\cdot \cancel{AP}\cdot QC + CP\cdot \cancel{AP}\cdot BQ$

$\Longleftrightarrow PQ\cdot BC \leq BP\cdot QC + CP\cdot BQ$

$\dfrac{BP}{QC}=\dfrac{AP}{AC}$

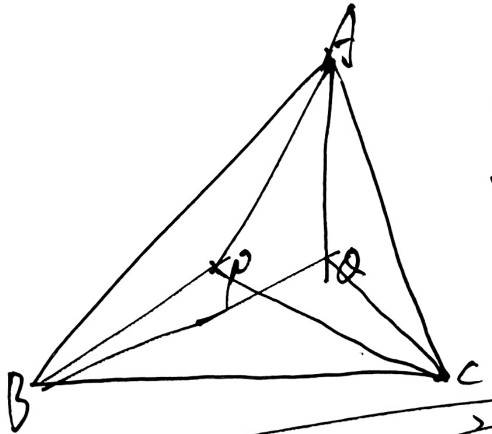
$\boxed{BP\cdot AC = AP\cdot QC}$

$AP^2\cdot BC + BP^2\cdot AC + CP^2\cdot AB$

何时取等号？P是内心.

$$\frac{AP\cdot AQ}{AB\cdot AC}+\cdots+\cdots=1$$

$$\boxed{AP\cdot AQ}\cdot BC+BP\cdot BQ\cdot AC+CP\cdot CQ\cdot AB=AB\cdot BC\cdot CA$$

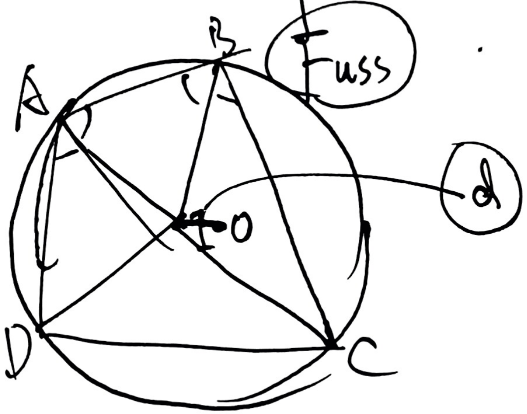


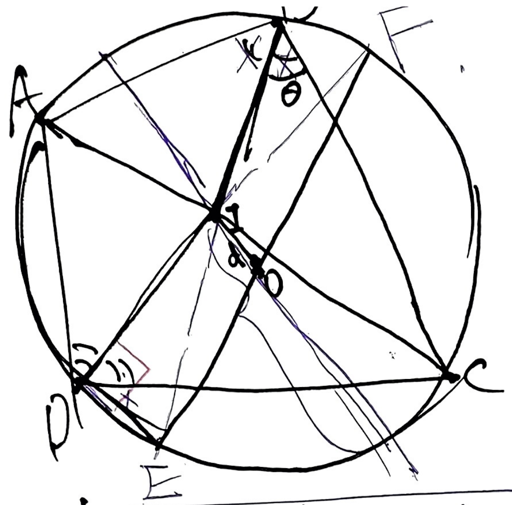
内心 欧拉定理 $\frac{1}{R+d}+\frac{1}{R-d}=\frac{1}{r}$

$(R^2-d^2=2Rr)$

外

富斯定理 $\frac{1}{(R+d)^2}+\frac{1}{(R-d)^2}=\frac{1}{r^2}$





△IEF中，IO为中线

$$IO^2+OE^2=\frac{1}{2}(IE^2+IF^2)$$

$$2(d^2+R^2)=\frac{(R^2-d^2)^2}{IB^2}+\frac{(R^2-d^2)^2}{ID^2}$$

$$2(d^2+R^2)=(R^2-d^2)^2\cdot\frac{1}{r^2}$$

由※

$$2(d^2+R^2)r^2=(R+d)^2(R-d)^2$$

$$\frac{2(d^2+R^2)}{(R+d)^2(R-d)^2}=\frac{1}{r^2}\iff\frac{(R+d)^2+(R-d)^2}{(R+d)^2(R-d)^2}=\frac{1}{r^2}$$

$$\iff\frac{1}{(R+d)^2}+\frac{1}{(R-d)^2}=\frac{1}{r^2}$$

$$IE\cdot IB=IF\cdot ID=R^2-d^2$$

$$IE=\frac{R^2-d^2}{IB}\qquad IF=\frac{R^2-d^2}{ID}$$

$$\frac{r}{IB}=\sin\theta\qquad\frac{r}{ID}=\sin(90^\circ-\theta)=\cos\theta$$

$$\frac{r^2}{IB^2}+\frac{r^2}{ID^2}=1\Rightarrow\frac{1}{IB^2}+\frac{1}{ID^2}=\frac{1}{r^2}$$

证

$$\boxed{\frac{1}{(R+d)^2}+\frac{1}{(R-d)^2}=\frac{1}{r^2}}$$

与d有关的东西？

圆幂：$\boxed{R^2-d^2}$

点I对⊙O的幂

$$\left(\frac{r}{R+d}\right)^2+\left(\frac{r}{R-d}\right)^2=1\quad ?$$

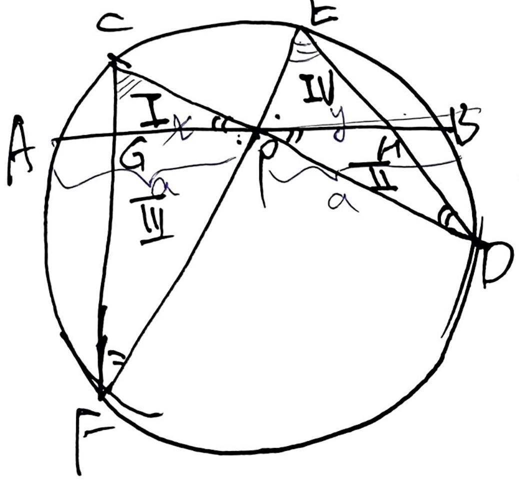
蝴蝶定理的证明

条件 $PA=PB$

结论 $PG=PH$

1、面积法

（张景中）



$$1=\frac{S_{I}}{S_{II}}\cdot\frac{S_{II}}{S_{III}}\cdot\frac{S_{III}}{S_{IV}}\cdot\frac{S_{IV}}{S_{I}}$$

$$=\frac{\cancel{PC}\cdot PG}{\cancel{PD}\cdot PH}\cdot\frac{\cancel{PD}\cdot HD}{\cancel{PF}\cdot GF}\cdot\frac{PG\cdot\cancel{PF}}{PH\cdot\cancel{PE}}\cdot\frac{\cancel{PE}\cdot HE}{\cancel{PC}\cdot GC}$$

$$=\frac{PG^2}{PH^2}\cdot\frac{HD\cdot HE}{GC\cdot GF}$$

要8日记

$$\frac{PH^2}{PG^2}=\frac{HD\cdot HE}{GC\cdot GF}=\frac{HB\cdot HA}{GA\cdot GB}$$

$$\frac{y^2}{x^2}=\cancel{\frac{(a-y)(a+y)}{(a-x)(a+x)}}=\frac{a^2-y^2}{a^2-x^2}\left(=\frac{y^2+a^2-y^2}{x^2+a^2-x^2}=1\right)$$

$$a^2y^2-\cancel{x^2y^2}=a^2x^2-\cancel{x^2y^2}$$

$$a^2y=a^2x$$

$$\underline{y=x}$$

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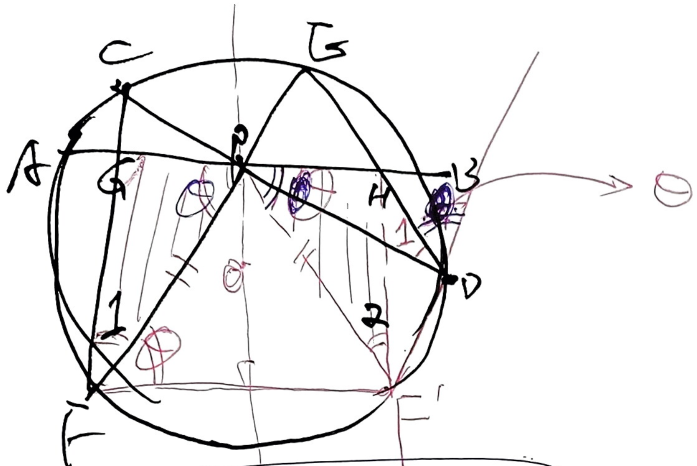
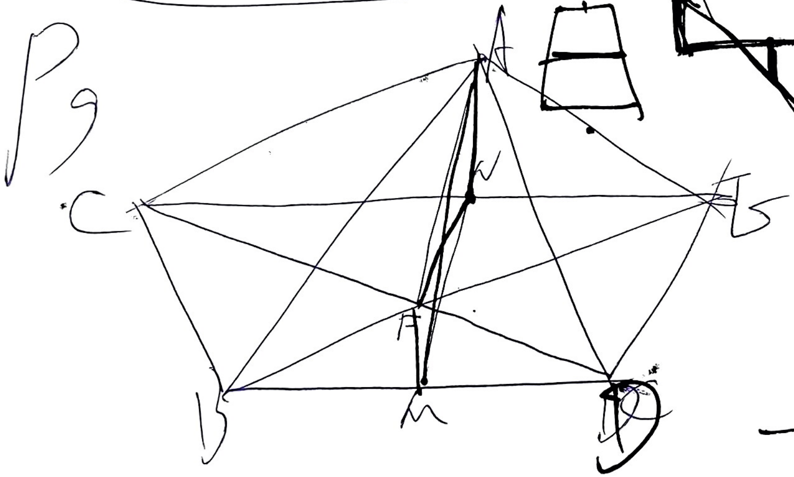


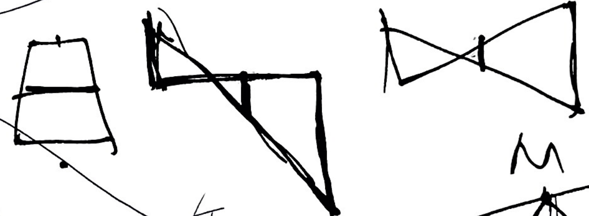
$AF \parallel MN \iff S_{\triangle AFM} = S_{\triangle AFN}$

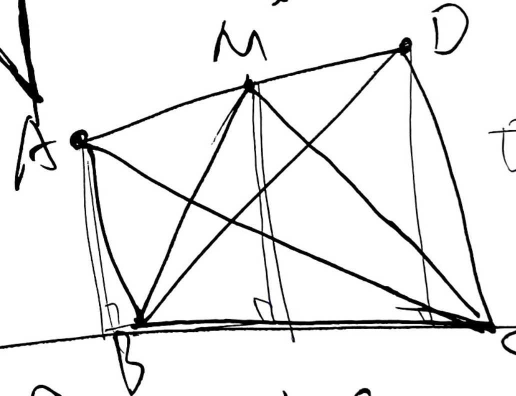
消M (B, D)

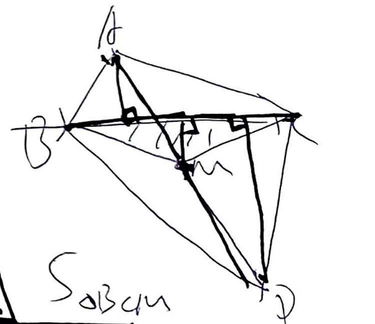
$S_{\triangle AFM} = \frac{1}{2}(S_{\triangle AFD} - S_{\triangle AFB})$

证 P、H、D、F' 共圆



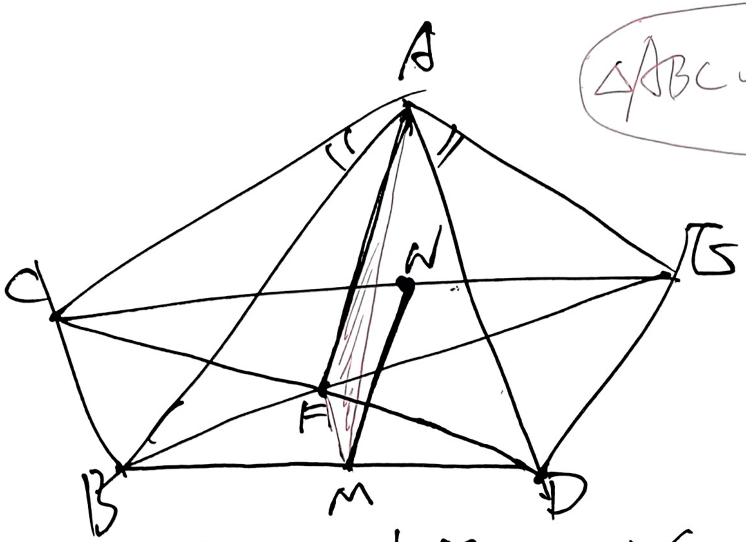






$S_{\triangle BCM} = \frac{1}{2}(S_{\triangle BCD} - S_{\triangle BCA})$

$S_{\triangle BCM} = \frac{1}{2}(S_{\triangle BCA} + S_{\triangle BCD})$



$\triangle ABC \sim \triangle ADE$

$\Downarrow$

$$\frac{AB}{AC} = \frac{AD}{AE}$$

$$\Longrightarrow AB \cdot AE = AC \cdot AD$$

20

$$S_{\triangle AFM} = \frac{1}{2}\left(S_{\triangle AFD} - \frac{1}{2} S_{\triangle AFB}\right)$$

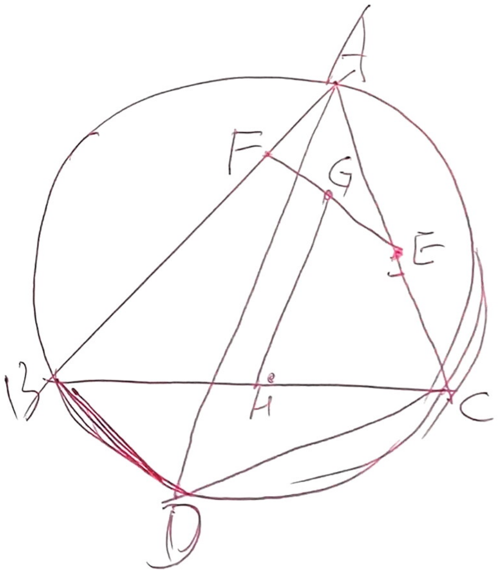
$$S_{\triangle AFN} = \frac{1}{2}\left(S_{\triangle AFE} - S_{\triangle AFC}\right)$$

只需证 $S_{\triangle AFD} - S_{\triangle AFB} = S_{\triangle AFE} - S_{\triangle AFC}$

$$\Longleftrightarrow \underline{S_{\triangle AFD} + S_{\triangle AFC}} = \underline{S_{\triangle AFE} + S_{\triangle AFB}}$$

$$\Longleftrightarrow S_{\triangle ACD} = S_{\triangle ABE}$$

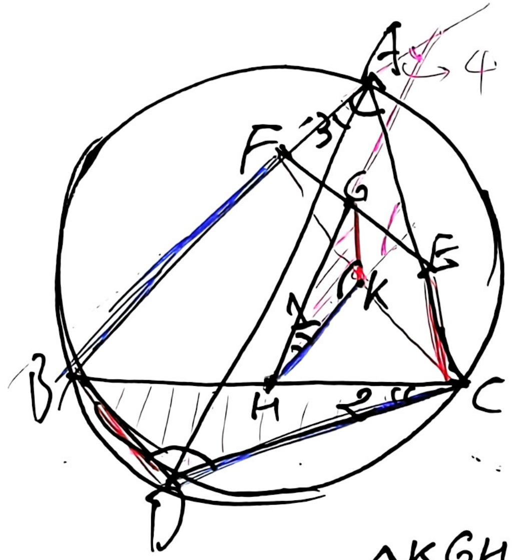
$$\Longleftrightarrow \frac{1}{2} AC \cdot AD \sin\cancel{\angle CAD} = \frac{1}{2} AB \cdot AE \sin\cancel{\angle BAE}$$

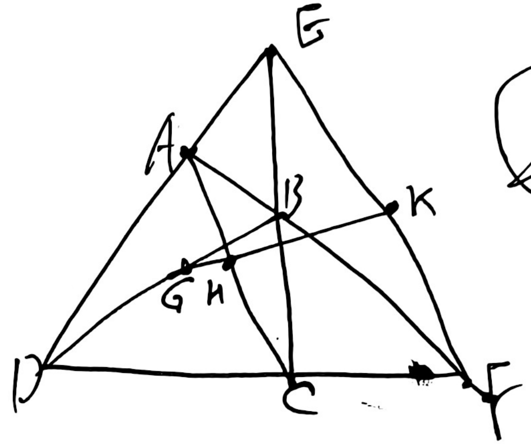


CE = BD　CD = BF

G.H 为中点

故 GH ∥ AD





牛顿线

消点

巧用中点

倍长中线

$\triangle KGH \sim \triangle DBC$

SAS?

$\Rightarrow \angle 1=\angle 2=\angle 3$

$HK \parallel AB$

$\boxed{HG \parallel AD}$

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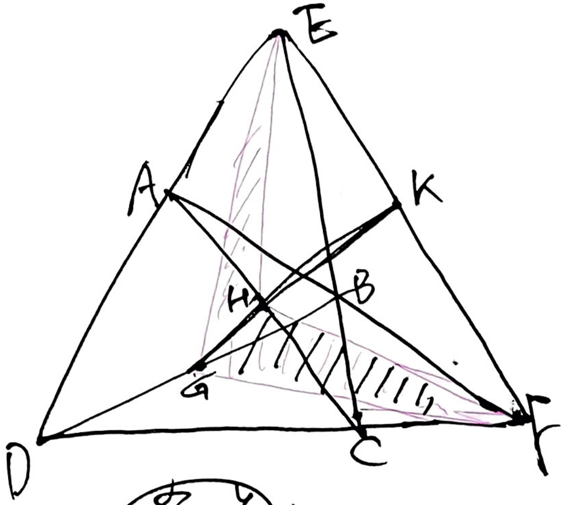
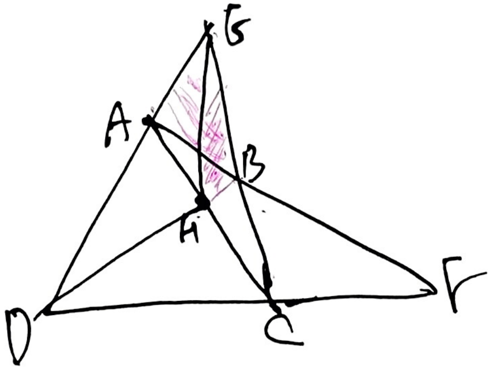
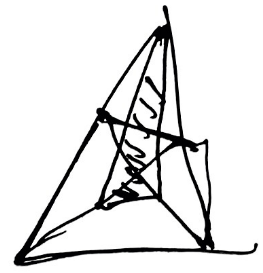






消K点

证 $\Longleftrightarrow S_{\triangle EHG} = S_{\triangle FHG}$

消G

$\Longleftrightarrow \frac{1}{2}(S_{\triangle EHD} - S_{\triangle EHB}) = \frac{1}{2}(S_{\triangle FHD} - S_{\triangle FHB})$

消H.

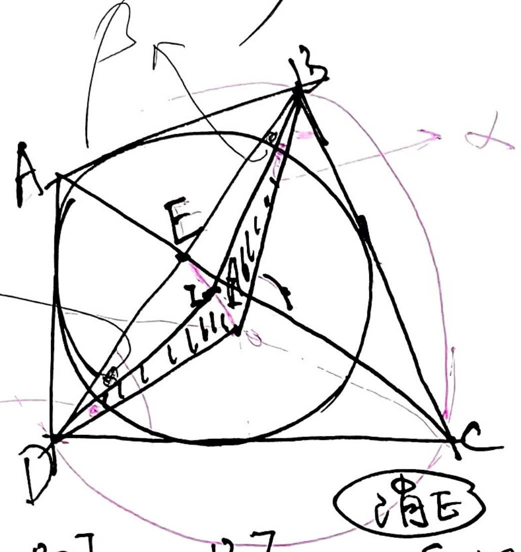
$S_{\triangle EDH} = \frac{1}{2} S_{\triangle CDG}$ $\quad S_{\triangle EHB} = \frac{1}{2} S_{\triangle ABG}$

左边 $= \frac{1}{2}\left(\frac{1}{2} S_{\triangle CDG} - \frac{1}{2} S_{\triangle ABG}\right)$

$= \frac{1}{4} S_{ABCD}$

同理 右边 $= \frac{1}{4} S_{ABCD}$

双心四边形



$\angle OBI = \angle OBI - \angle OBO$

$= 90 - \angle CDI - (90 - \angle BDC)$

$= \angle BDC - \angle CDI$

$= \angle IDB$

$\Longleftrightarrow \dfrac{\dfrac{\sin\frac{A+B}{2}}{\sin\frac{A}{2}\sin\frac{B}{2}}\cdot\dfrac{\sin\frac{B+C}{2}}{\sin\frac{B}{2}\sin\frac{C}{2}}}{\dfrac{\sin\frac{A+D}{2}}{\sin\frac{A}{2}\sin\frac{D}{2}}\cdot\dfrac{\sin\frac{C+D}{2}}{\sin\frac{D}{2}\sin\frac{C}{2}}} = \dfrac{\sin^2\frac{D}{2}}{\sin^2\frac{B}{2}}$

$A+B+C+D=360$

对称分式为何

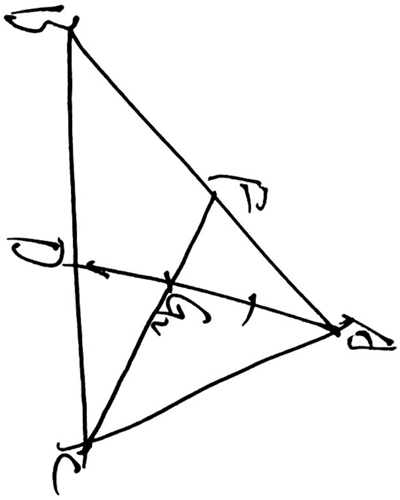
$$\frac{S_{\triangle BOI}}{S_{\triangle DOI}} = \frac{BE}{DE} = \frac{S_{\triangle ABC}}{S_{\triangle DAC}} = \frac{AB\cdot BC}{AD\cdot DC}$$

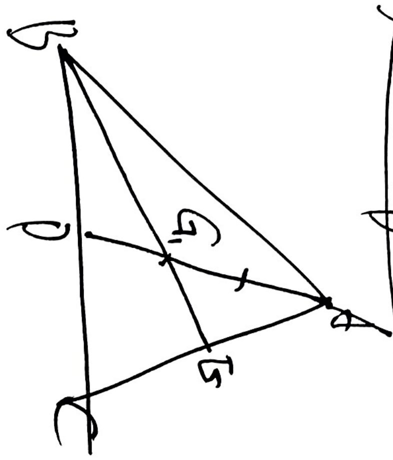
$$\frac{BI\cdot BO\sin\angle OBI}{DI\cdot DO\cdot\sin\angle ODI} = \frac{BI\cdot\sin\alpha}{DI\cdot\sin\beta} = \frac{BI}{DI}\cdot\frac{BI}{DI} = \frac{BI^2}{DI^2}$$

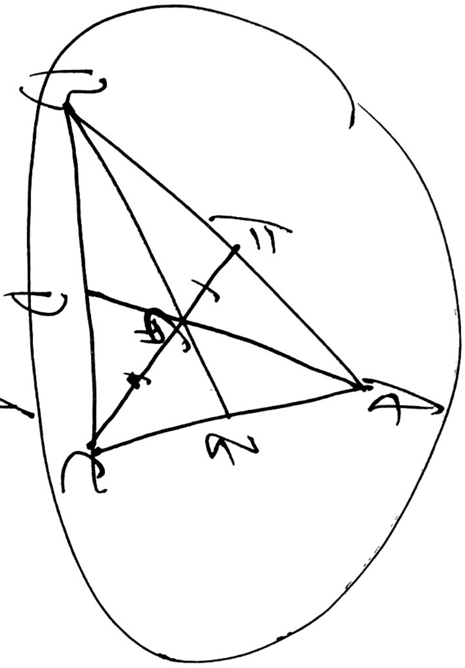
转化为证 $\dfrac{AB\cdot BC}{AD\cdot DC} = \dfrac{BI^2}{DI^2} \Longleftrightarrow \dfrac{(\cot\frac{A}{2}+\cot\frac{B}{2})(\cot\frac{B}{2}+\cot\frac{C}{2})}{(\cot\frac{A}{2}+\cot\frac{D}{2})(\cot\frac{D}{2}+\cot\frac{C}{2})} = \dfrac{\frac{1}{\sin^2\frac{B}{2}}}{\frac{1}{\sin^2\frac{D}{2}}}$

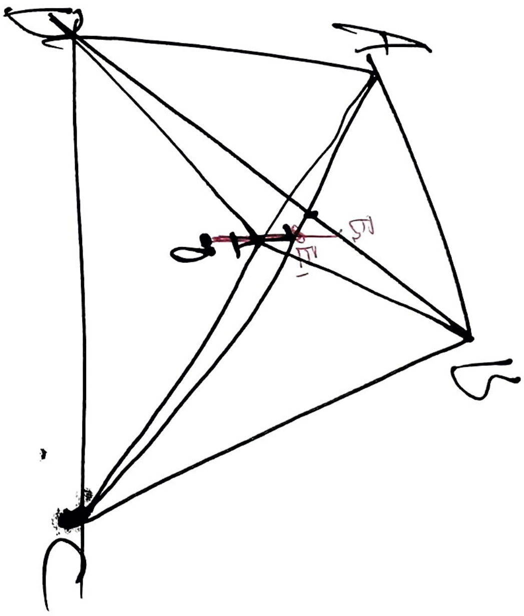
将其化为结论

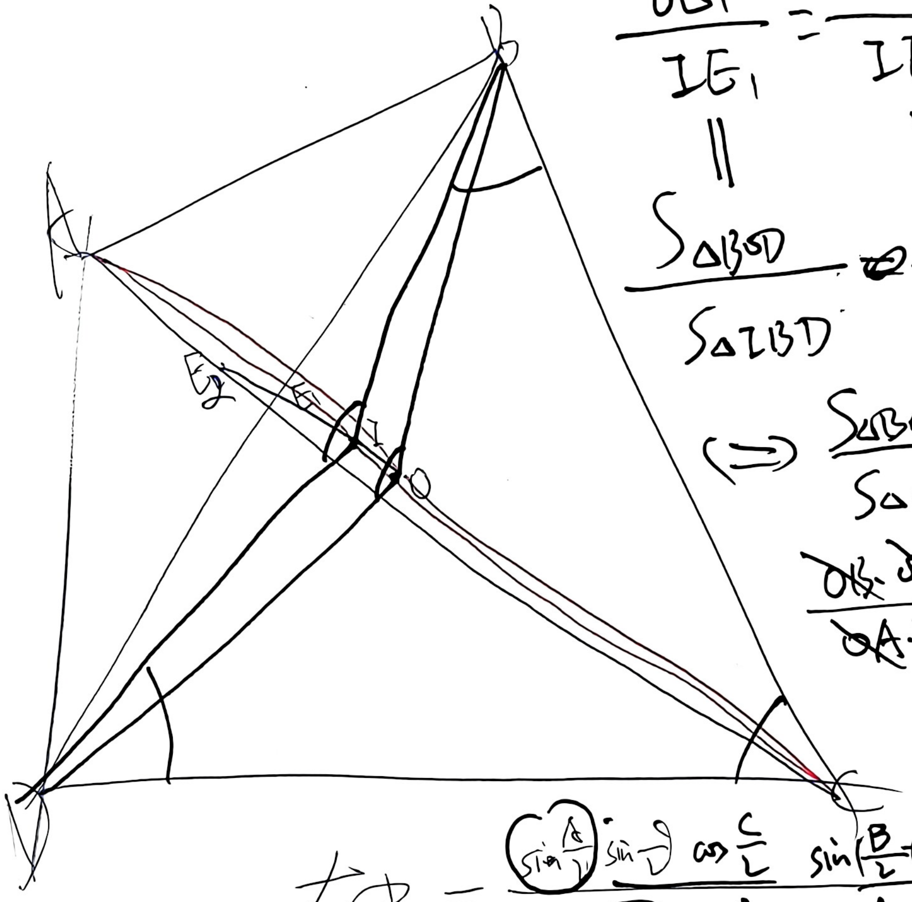
同一法











$$\frac{OE_1}{IE_1} = \frac{OE_2}{IE_2}$$

$$\parallel \qquad\qquad \parallel$$

$$\frac{S_{\triangle BOD}}{S_{\triangle IBD}} = \frac{S_{\triangle OAC}}{S_{\triangle IAC}}$$

$$\iff \frac{S_{\triangle BOD}}{S_{\triangle OAC}} = \frac{S_{\triangle IBD}}{S_{\triangle IAC}}$$

$$\frac{\cancel{OB\cdot OD}\sin 2C}{\cancel{OA\cdot OC}\sin 2B} = \frac{IB\cdot ID\sin\left(\frac{B}{2}+\frac{D}{2}+C\right)}{IA\cdot IC\sin\left(\frac{A}{2}+\frac{C}{2}+B\right)}$$

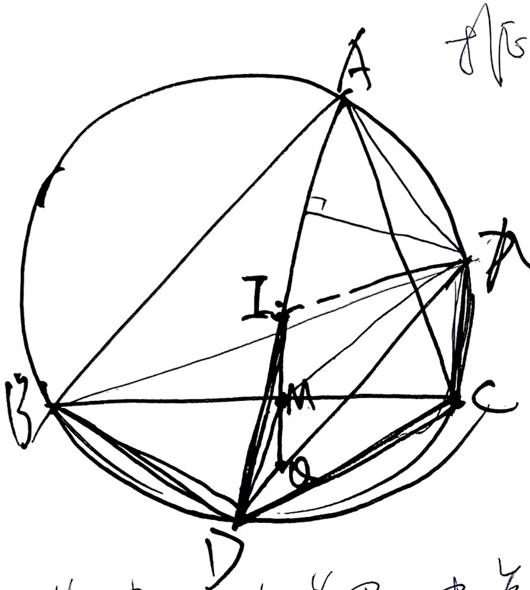
$$= \frac{\frac{\cancel{r}}{\sin\frac{B}{2}}\cdot\frac{\cancel{r}}{\sin\frac{D}{2}}\sin\left(\frac{B}{2}+\frac{D}{2}+C\right)}{\frac{\cancel{r}}{\sin\frac{A}{2}}\cdot\frac{\cancel{r}}{\sin\frac{C}{2}}\sin\left(\frac{A}{2}+\frac{C}{2}+B\right)}$$

$$右边 = \frac{\sin\frac{A}{2}\,\sin\frac{C}{2}\,\cos\frac{C}{2}\,\sin\left(\frac{B}{2}+\frac{D}{2}+C\right)}{\sin\frac{B}{2}\,\sin\frac{D}{2}\,\cos\frac{B}{2}\,\sin\left(\frac{A}{2}+\frac{C}{2}+B\right)}$$

$$\iff \frac{\cancel{2\sin C}\cos C}{\cancel{2\sin B}\cos B} = \frac{\cancel{\sin C}}{\cancel{\sin B}}\cdot\frac{\sin\left(\frac{B}{2}+\frac{D}{2}+C\right)}{\sin\left(\frac{A}{2}+\frac{C}{2}+B\right)} \quad \left(\iff \frac{\sin(\delta+C)}{\sin(\delta+B)} = \frac{\cos C}{\cos B}\right)$$

67:

求证 $BN = AN + CN$



I为内心 M为BC中点

$IM = MQ$

$DI = BD = CD$

面积法

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$\triangle IDN$

$\triangle DMN$ //

$S_{\triangle DMN} = \frac{1}{2}(S_{\triangle DNB} - S_{\triangle DNC})$

$S_{\triangle IDN} = 2S_{\triangle DMN}$

$S_{\triangle IDN} = S_{\triangle DNB} - S_{\triangle DNC}$

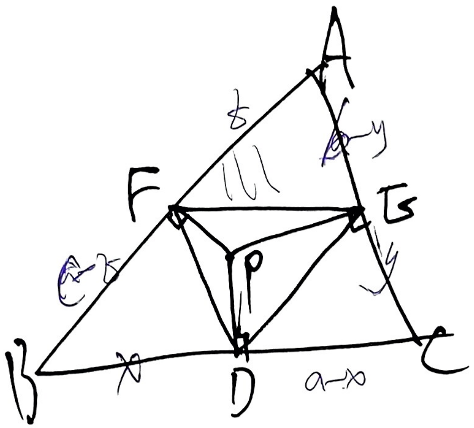
$= \frac{1}{2} BD \cdot BN \sin\angle NBD - \frac{1}{2} CN \cdot CD \sin\angle DCN$

互补

$S_{\triangle IDN} = \frac{1}{2} DI \cdot h = \frac{1}{2} DI \cdot AN \sin\angle DAN$

$\therefore AN = BN - CN$

$\therefore BN = AN + CN.$



求证: $S_{\triangle DEF} \le \frac{1}{4} S_{\triangle ABC}$

$$\frac{S_{\triangle DEF}}{S_{\triangle ABC}} \le \frac{1}{4}$$

即 $\frac{S_{\triangle AEF}}{S_{\triangle ABC}} - \frac{S_{\triangle BFD}}{S_{\triangle ABC}} - \frac{S_{\triangle CDE}}{S_{\triangle ABC}}$

只证 $\frac{z(b-y)}{bc} + \frac{(c-z)x}{ca} + \frac{(a-x)y}{ab} \ge \frac{3}{4}$

已知 $x^2+y^2+z^2$

$= (a-x)^2+(b-y)^2+(c-z)^2$