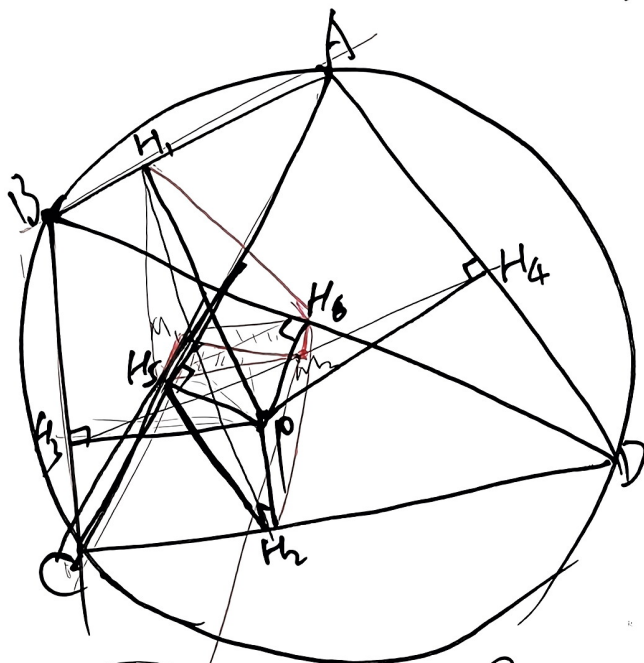


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$$\begin{aligned}
 & \frac{1}{2} (S_{\Delta H_6 H_1 H_3} - S_{\Delta H_6 H_1 H_4}) \\
 & \Rightarrow S_{\Delta H_6 H_1 H_3} - S_{\Delta H_6 H_1 H_4} = S_{\Delta H_6 H_1 H_3} + S_{\Delta H_6 H_1 H_4} \\
 & = S_{\Delta H_1 H_5 H_6} - S_{\Delta H_1 H_5 H_3} - S_{\Delta H_5 H_2 H_4} + S_{\Delta H_5 H_2 H_3} \\
 & \Rightarrow S_{\Delta H_6 H_1 H_3} + S_{\Delta H_6 H_1 H_4} + S_{\Delta H_1 H_5 H_3} + S_{\Delta H_5 H_2 H_4} \\
 & = S_{\Delta H_1 H_5 H_6} + S_{\Delta H_5 H_2 H_3} + S_{\Delta H_6 H_1 H_4} + S_{\Delta H_6 H_1 H_3}
 \end{aligned}$$

$$\frac{1}{2} (S_{\Delta H_6 H_1 H_3} - S_{\Delta H_6 H_1 H_4})$$

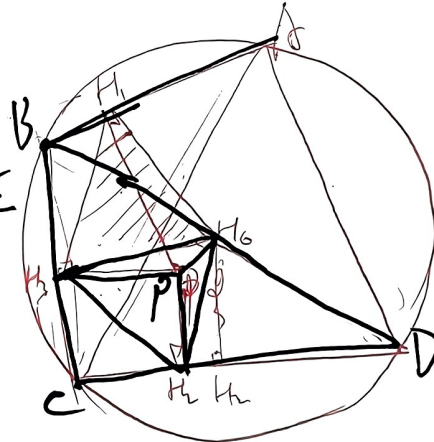
$$= \frac{1}{2} (S_{\Delta H_1 H_5 H_6} - S_{\Delta H_1 H_5 H_3}) = \frac{1}{2} (S_{\Delta H_5 H_2 H_4} - S_{\Delta H_5 H_2 H_3})$$

$$(\Rightarrow) S_{\Delta H_6 H_1 H_3} - S_{\Delta H_6 H_1 H_4} = S_{\Delta H_6 H_1 H_3} + S_{\Delta H_6 H_1 H_4}$$

$$= S_{\Delta H_1 H_5 H_6} - S_{\Delta H_1 H_5 H_3} - S_{\Delta H_5 H_2 H_4} + S_{\Delta H_5 H_2 H_3}$$

$$(\Rightarrow) S_{\Delta H_6 H_1 H_3} + S_{\Delta H_6 H_1 H_4} + S_{\Delta H_1 H_5 H_3} + S_{\Delta H_5 H_2 H_4}$$

$$= S_{\Delta H_1 H_5 H_6} + S_{\Delta H_5 H_2 H_3} + S_{\Delta H_6 H_1 H_4} + S_{\Delta H_6 H_1 H_3}$$



$$S_{\Delta H_2 H_6 H_3} = \frac{R^2 - OP^2}{4R^2} S_{\Delta BCD}$$

$$S_{\Delta H_1 H_5 H_3} = \frac{R^2 - OP^2}{4R^2} S_{\Delta ABC}$$

$$S_{\Delta H_6 H_1 H_4} = \frac{R^2 - OP^2}{4R^2} S_{\Delta ACD}$$

$$S_{\Delta H_5 H_2 H_4} = \frac{R^2 - OP^2}{4R^2} S_{\Delta ADB}$$

$$S_{\Delta H_1 H_6 H_4} = \frac{R^2 - OP^2}{4R^2} S_{\Delta ABD}$$

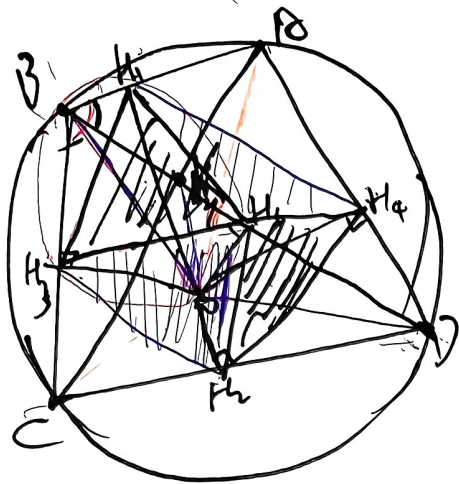
$$(\Rightarrow) S_{\Delta H_6 H_1 H_3} + S_{\Delta H_2 H_6 H_3} = S_{\Delta H_1 H_5 H_3} + S_{\Delta H_5 H_2 H_4}$$



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$$\frac{AD \cdot CD \cdot AC}{4R} = S_{\triangle ACD}$$

$$S_{\triangle H_1 H_3 H_4} = \frac{1}{2} AP^2 \cdot \frac{S_{\triangle BCD}}{2R^2}$$

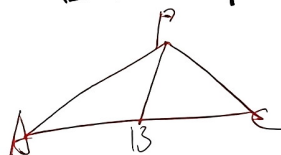
$$S_{\triangle H_5 H_2 H_3} = \frac{1}{2} CP^2 \cdot \frac{S_{\triangle ABD}}{2R^2}$$

两式相加

$$\frac{BP^2 \cdot S_{\triangle ACD}}{S_{\triangle BCD}} + \frac{DP^2 \cdot S_{\triangle ABD}}{S_{\triangle ABD}} = \frac{AP^2 \cdot S_{\triangle BCD}}{S_{\triangle BCD}} + \frac{CP^2 \cdot S_{\triangle ABD}}{S_{\triangle ABD}}$$

$$\Rightarrow BP^2 \cdot \frac{DK}{BD} + DP^2 \cdot \frac{BK}{BD} = AP^2 \cdot \frac{CK}{AC} + CP^2 \cdot \frac{AK}{AC}$$

$$\Rightarrow \frac{BP^2 \cdot DK + DP^2 \cdot BK}{BD} = \frac{AP^2 \cdot CK + CP^2 \cdot AK}{AC}$$



$$PA^2 \cdot BC + PC^2 \cdot AB$$

$$= PB^2 \cdot AC + AB \cdot BC \cdot AC$$

$$\frac{PK^2 \cdot BD + BK \cdot DK \cdot BD}{BD} = \frac{PK^2 \cdot AC + AK \cdot CK \cdot AC}{AC}$$

$$\Rightarrow PK^2 + BK \cdot DK = PK^2 + AK \cdot CK$$

$$S_{\triangle H_1 H_3 H_6} = \frac{1}{2} H_1 H_6 \cdot H_3 H_6 \cdot \sin \angle H_1 H_6 H_3$$

$$= \frac{1}{2} BP^2 \sin \angle ABD \cdot \sin B$$

$$= \frac{1}{2} BP^2 \sin \angle ABD \sin \angle CBD \cdot \sin B$$

$$= \frac{1}{2} BP^2 \cdot \frac{AD}{2R} \cdot \frac{CD}{2R} \cdot \frac{AC}{2R}$$

$$= \frac{1}{2} BP^2 \cdot \frac{S_{\triangle ACD}}{2R^2}$$

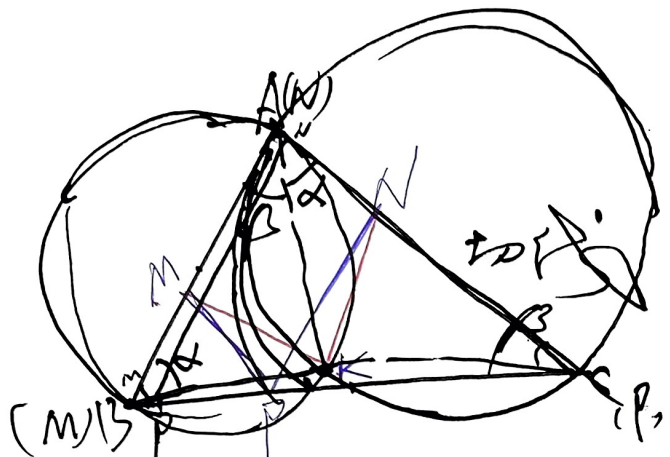
$$|2P| S_{\triangle H_2 H_6 H_4} = \frac{1}{2} DP^2 \cdot \frac{S_{\triangle ABC}}{2R^2}$$



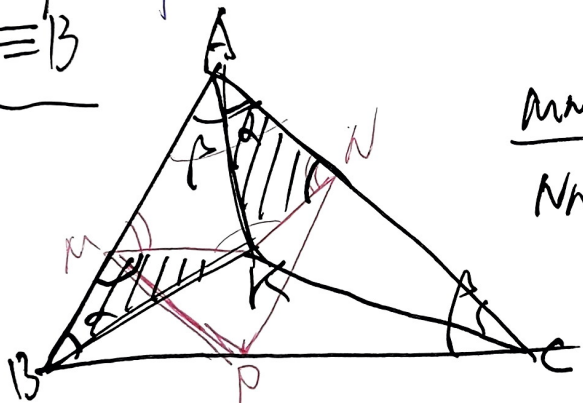
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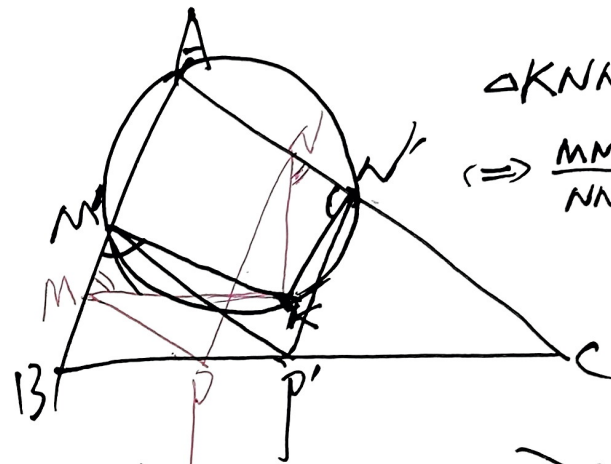
$P \equiv B$



$\triangle ABK \sim \triangle CAK$

$$\frac{AM}{BM} = \frac{CP}{BP} = \frac{CN}{AN}$$

M, N 对径点



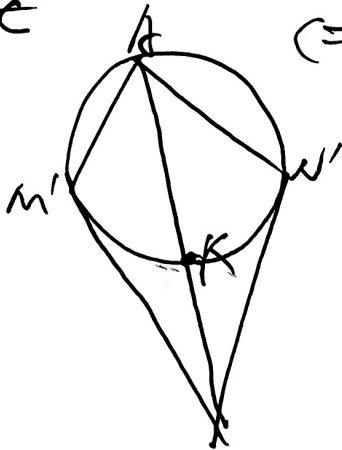
$\triangle KNN' \sim \triangle KMM'$

$$\Rightarrow \frac{MM'}{NN'} = \frac{M'K}{N'K}$$

$$\frac{MM'}{NN'} = \frac{MM'}{PP'} \cdot \frac{PP'}{NN'} = \frac{BM'}{BP'} \cdot \frac{CP'}{CN'} = \frac{BM'}{CN'}$$

$$= \frac{AM'}{AN'} \quad \text{只需证} \quad \frac{AM'}{AN'} = \frac{M'K}{N'K}$$

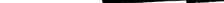
$$\Rightarrow AM' \cdot N'K = M'K \cdot AN'$$

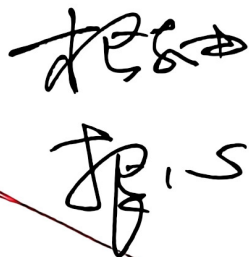


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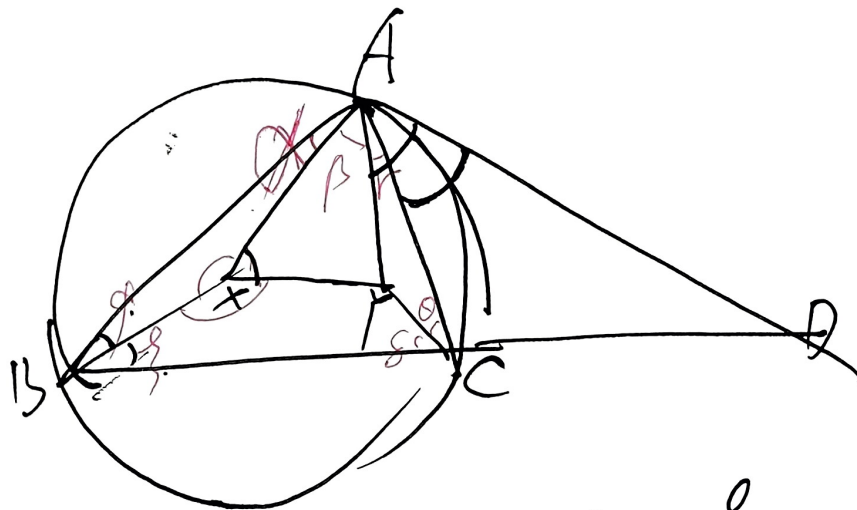


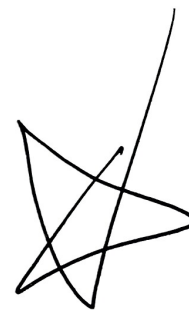

$$\ominus AX^T = \frac{w_2}{w_2} \} \text{ result: } X^T$$

AD Sw, 22p

Hand-drawn geometric diagram illustrating a construction involving a circle and several points. The diagram shows a circle with center  $O$  and a point  $A$  on its circumference. A line segment  $AD$  is drawn from  $A$  to a point  $D$  outside the circle. A line segment  $AB$  is drawn from  $A$  to a point  $B$  on the circle. A line segment  $AC$  is drawn from  $A$  to a point  $C$  on the circle. A line segment  $BC$  is drawn from  $B$  to  $C$ . A line segment  $BD$  is drawn from  $B$  to  $D$ . A line segment  $CD$  is drawn from  $C$  to  $D$ . A line segment  $AX$  is drawn from  $A$  to a point  $X$  on  $AD$ . A line segment  $BX$  is drawn from  $B$  to  $X$ . A line segment  $CX$  is drawn from  $C$  to  $X$ . A line segment  $DX$  is drawn from  $D$  to  $X$ . The diagram is labeled with various points and lines, and includes a handwritten note  $AX = w_2$ .



$$\begin{aligned} \angle AXB - \angle ACB &= \beta + \gamma + \delta \\ \angle AYC - \angle AXB &= \beta + \alpha + \delta \end{aligned} \Rightarrow \underline{\underline{\gamma + \delta = \alpha + \delta}}$$



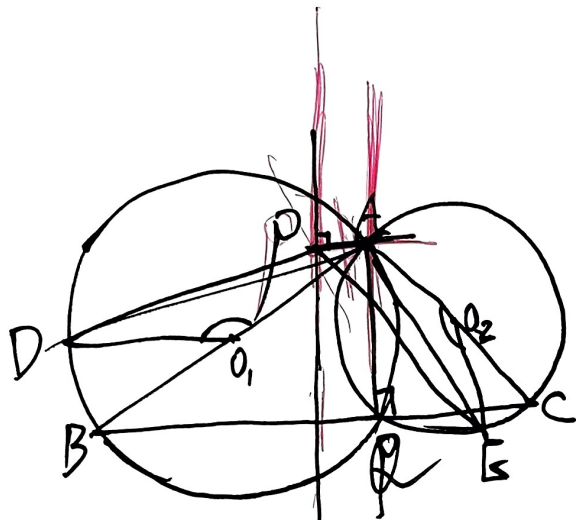
西  
56

$$\begin{aligned} \angle OAY &= \angle AXY = 360^\circ - \angle AXB - \angle BXY \\ &= 360^\circ - (180^\circ - \alpha - \gamma) - (180^\circ - \delta) \\ &= \alpha + \gamma + \delta \end{aligned}$$

即

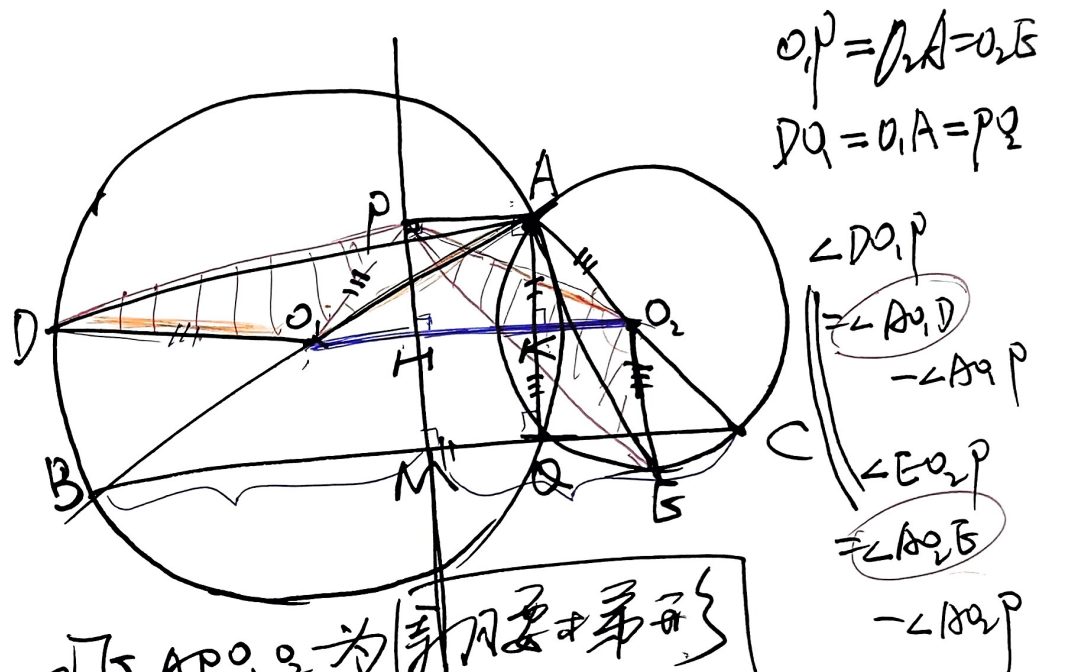
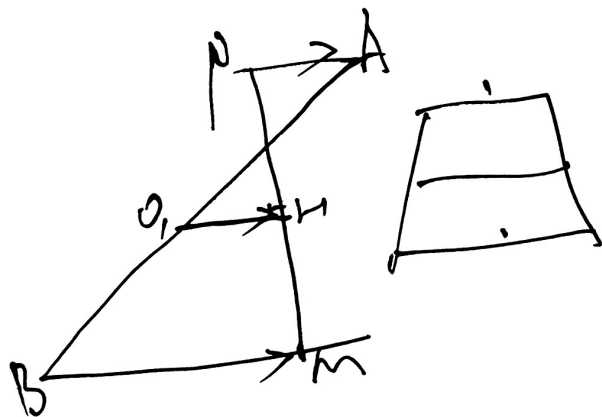
$$\begin{aligned} \gamma + \delta &= \alpha + \gamma + \delta \\ \Rightarrow \underline{\underline{\gamma + \delta = \alpha + \delta}} \end{aligned}$$





李瑞生 (A)

只靠  $PD = PT$



√6 AP<sub>0.2</sub> 为新月型第 3 号

$$\sqrt{6} \Delta \text{PHO} \cong \Delta \text{PKO}_2$$

$$pH = \Delta K \quad \Delta pH_1 = \Delta K_2 = 9.5$$

$$T - \sqrt{6} \, 0,14 = 0,14$$

$$0,14 = \frac{1}{2}$$

$$Q_K = \frac{1}{2} C_D$$

$$O_1H = \frac{1}{2} (3m - AP),$$

$$\text{只靠 } CQ = 13\text{m} - AP \Rightarrow CQ + AP = 13\text{m}$$

$$\Leftrightarrow 2Q + 10Q = 3m \Leftrightarrow 12Q = 3m \checkmark$$

$$Q_p = Q_A = Q_E$$

$$DQ = 0, A = PQ$$

$\angle DO, P$

$$= \angle AOD$$

$-\angle A_9 P$

$\angle EOP$

$$= \angle AOB$$

- 2/20/20

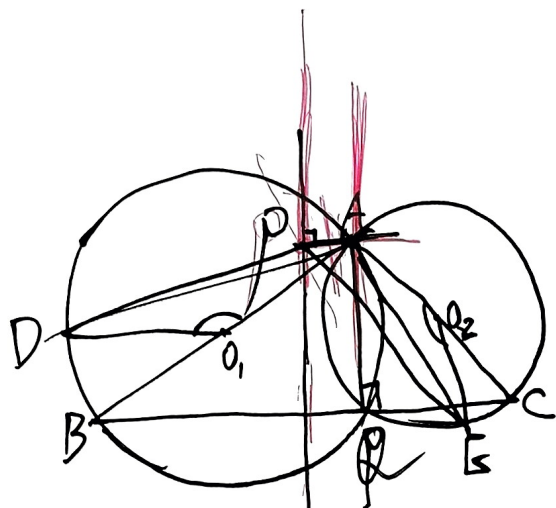
$\therefore \Delta DOP$

$$\approx \Delta p_{OE}$$

$$\therefore \underline{\underline{PD = PT}}$$

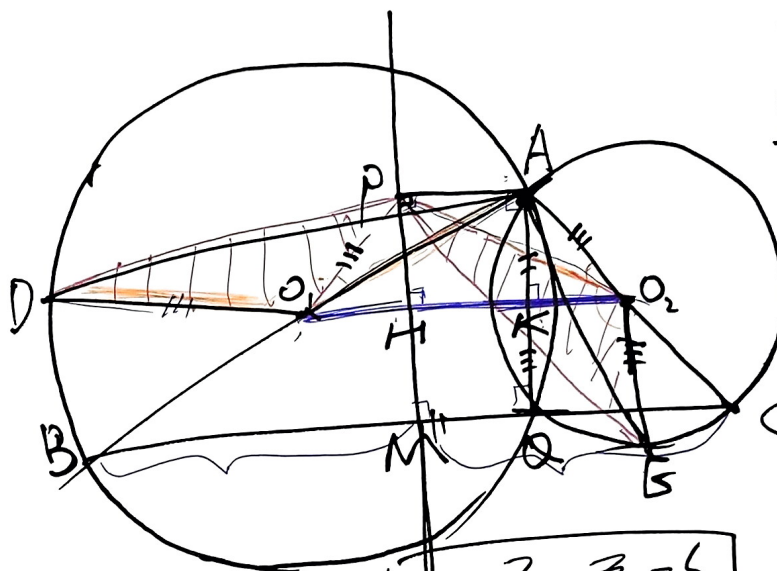
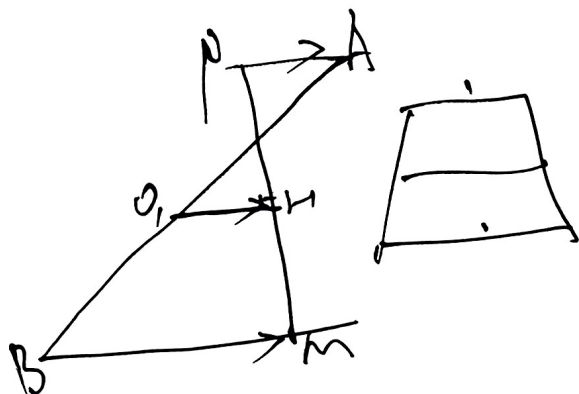


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在端点(A)

只需证  $PD = PE$



$\sqrt{5} AP, O_1, O_2$  为新月交点

$\sqrt{5} \triangle PHO_1 \cong \triangle PKO_2$

$PH = PK \quad \angle PHO_1 = \angle PKO_2 = 90^\circ$

$\sqrt{5} O_1H = O_2K$

$O_2K = \frac{1}{2} CO_2$

$O_1H = \frac{1}{2} (BM - AP)$

只需证  $CO_2 = BM - AP \Leftrightarrow CO_2 + AP = BM$

$\Leftrightarrow CO_2 + MO_2 = BM \Leftrightarrow CM = BM \checkmark$

$O_1P = O_2A = O_2E$   
 $DQ = O_1A = PQ$

$\angle DO_1P$   
 $= \angle AO_1D$   
 $= \angle AO_1P$   
 $\angle EO_2P$   
 $= \angle AO_2E$   
 $= \angle AO_2P$

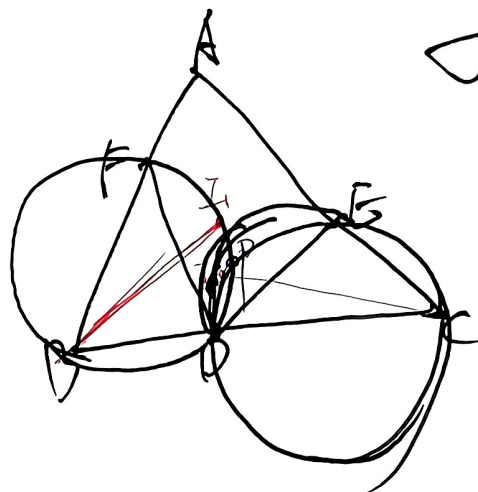
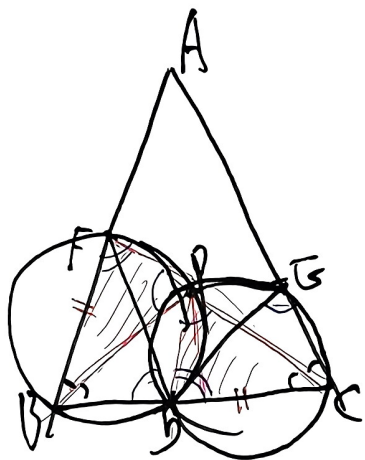
$\therefore \triangle DO_1P \cong \triangle EO_2P$   
 $\therefore PD = PE$

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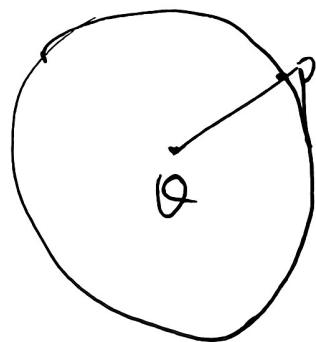
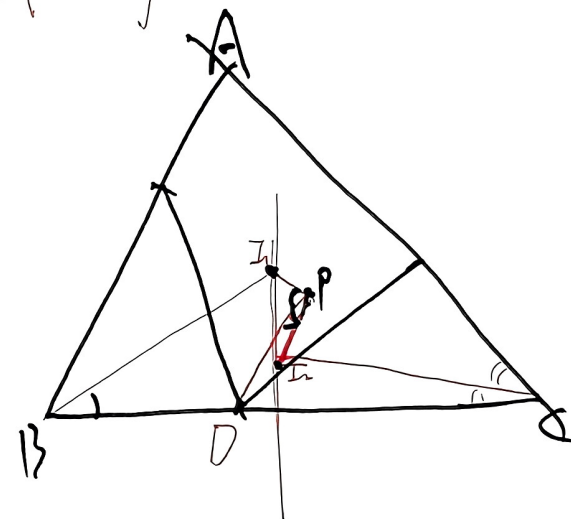
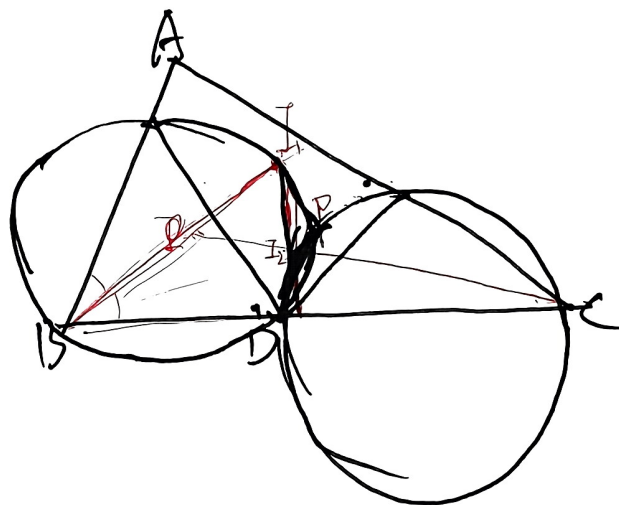
有向角

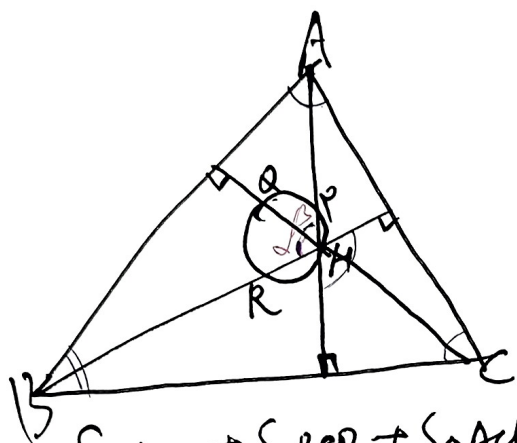


$\angle I_1 P I_2$  为定值

$$\triangle BFP \cong \triangle CBP$$

$$\Rightarrow FP = CP$$





$$S_{\triangle ABQ} + S_{\triangle BCP} + S_{\triangle ACR}$$

$$= S_{\triangle ABC}, \quad H \text{ 为垂心.}$$

证: H, P, Q, R 共圆.

$$S_{\triangle ABQ} + S_{\triangle BCP} + S_{\triangle ACR}$$

$$= S_{\triangle ABH} + S_{\triangle BCH} + S_{\triangle ACH}$$

$$(S_{\triangle BCP} - S_{\triangle BCH}) + (S_{\triangle ACR} - S_{\triangle ACH}) = (S_{\triangle ABH} - S_{\triangle ABQ})$$

$$\triangle BC \cdot PH + \triangle AC \cdot HR = \triangle AB \cdot HQ$$

$$\sin A \cos A = \sin C \cos C + \sin B \cos B - 2 \sin A \cos B \cos C$$

$$\Rightarrow \sin A (\cos A + 2 \cos B \cos C)$$

$$= \frac{1}{2} \sin 2A + \frac{1}{2} \sin 2B$$

$$\Rightarrow \sin A [-\cos(B+C) + 2 \cos B \cos C]$$

$$= \frac{1}{2} (\sin 2C + \sin 2B)$$

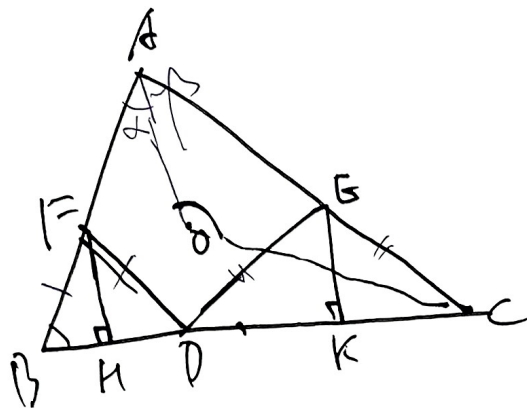
$$\Rightarrow \sin A \cos(B+C)$$

$$= \frac{1}{2} \cdot 2 \sin \frac{2C+2B}{2} \cos \frac{2C-2B}{2}$$

$$\text{证: } PH \sin A + HR \sin B = HQ \sin C$$

$$PH \sin A + HR \sin B = HQ \sin C$$





$$OA \sin A = AF \sin \beta + AE \sin \alpha$$

$$AF \sin(90^\circ - \beta) + AE \sin(90^\circ - \gamma)$$

$$\Rightarrow OA \sin A = AF \cos \beta + AE \cos \gamma$$

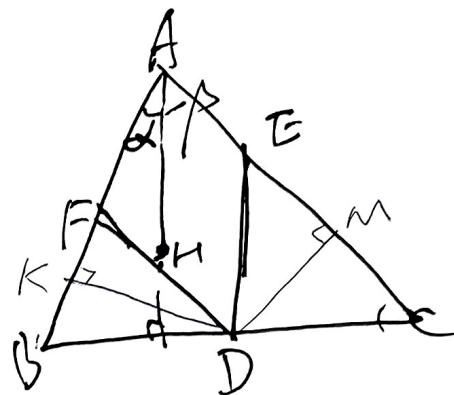
$$= (AB - BF) \cos \beta + (AC - CE) \cos \gamma$$

$$= AB \cos \beta + AC \cos \gamma - (BF \cos \beta + CE \cos \gamma)$$

$$\Rightarrow OA \sin A = \frac{AB \cos \beta + AC \cos \gamma}{BC} - \frac{1}{2}(BD + CD)$$

$$R \sin A = \frac{1}{2} \cdot 2R \sin A$$

$$= \frac{1}{2} \cdot BC$$



$$AH \sin A = AF \sin \beta + AE \sin \alpha$$

$$\Rightarrow 2R \cos A \sin A = (AB - 2BK) \cos \gamma + (AC - 2CM) \cos \beta$$

$$\Rightarrow BC \cos A = AB \cos \gamma + AC \cos \beta - 2BK \cos \gamma - 2CM \cos \beta$$

$$\Rightarrow \begin{aligned} & -2BK \cos \gamma \cos \beta - 2CM \cos \beta \cos \gamma \\ & = -2 \cos \beta \cos \gamma \cdot BC \end{aligned}$$

$$\Rightarrow BC \cos A = AB \cos \gamma + AC \cos \beta - 2BC \cos \beta \cos \gamma$$

