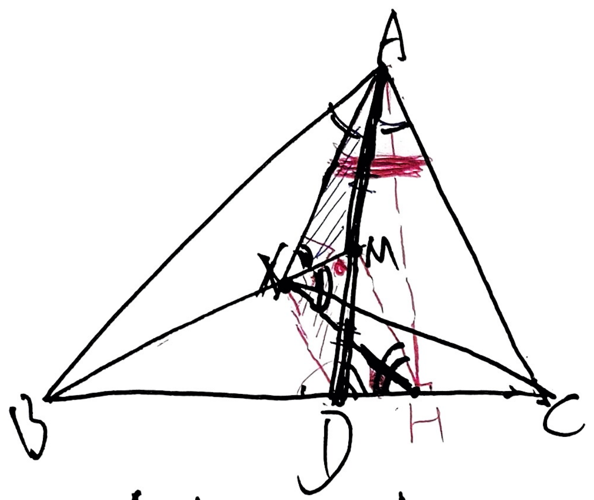


$\triangle AMX \sim \triangle BMA$

$MH^2 = DM^2 = AM^2 = MX \cdot MB$

$\triangle HMX \sim \triangle BMH$

$\Rightarrow \angle MXH = \angle MHD$

$MD = MH \Rightarrow \angle MDH = \angle MHD$

$\Rightarrow \angle MXH = \angle MDH$

$\Rightarrow X.M.H.D$ 四点共圆

下证 $A.X.H.C$ 共圆

$\angle DHX = \angle DMX = \angle ABM + \angle BAM$

$= \angle XAM + \angle CAD$

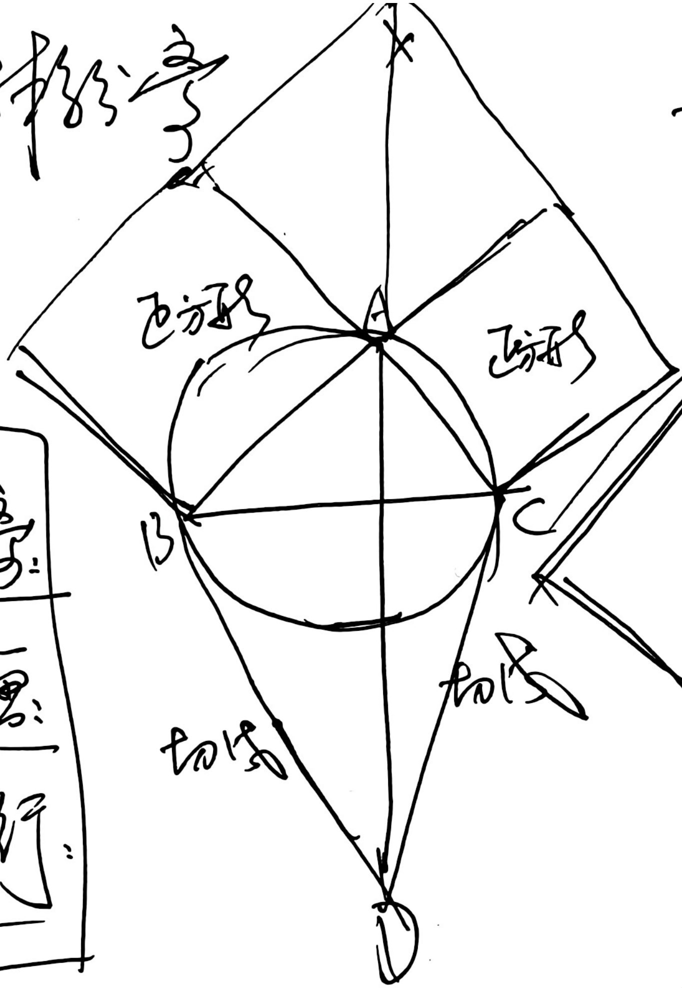
$= \angle CAX$

$\therefore A.X.H.C$ 共圆

$\therefore \angle AXC = \angle AHC = 90$

$\therefore AX \perp XC.$

对称分享



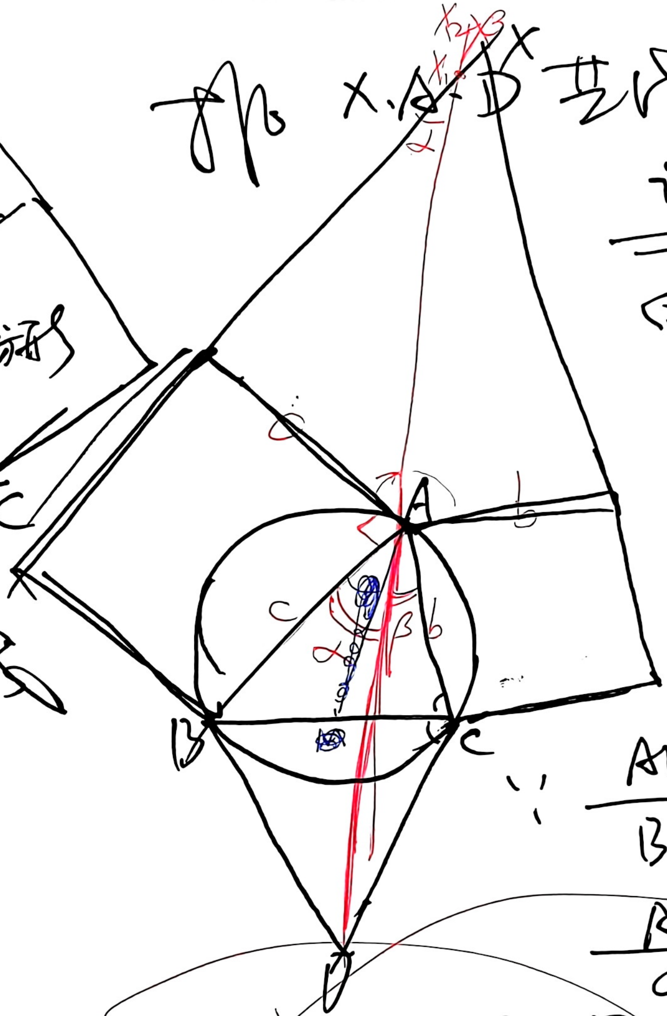
如:

分享:

剖要:

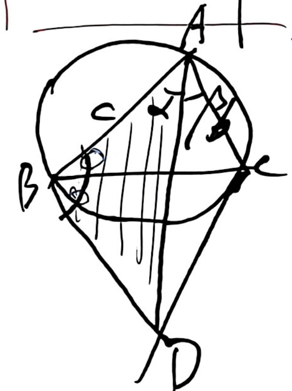
执行:

拆 X, A, D 共线



证明: $AX_1 = AX_2$

$\iff \frac{c}{\sin\alpha} = \frac{b}{\sin\beta}$



$\because \frac{AD}{BD} = \frac{\sin(B+A)}{\sin\alpha} = \frac{\sin C}{\sin\alpha}$

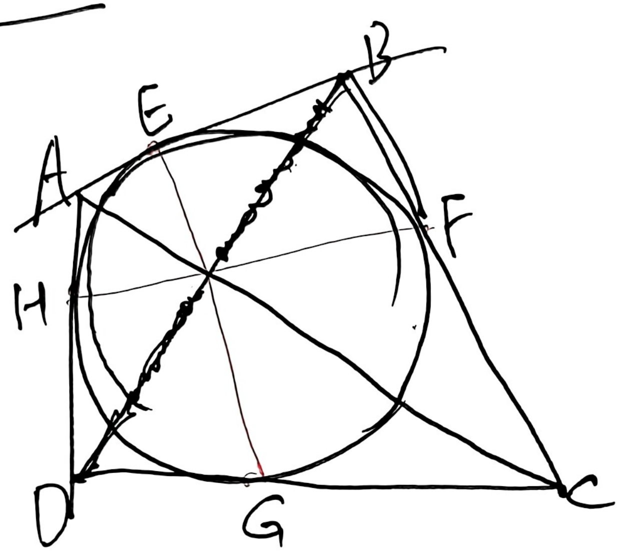
$\frac{AD}{CD} = \frac{\sin(A+C)}{\sin\beta} = \frac{\sin B}{\sin\beta}$

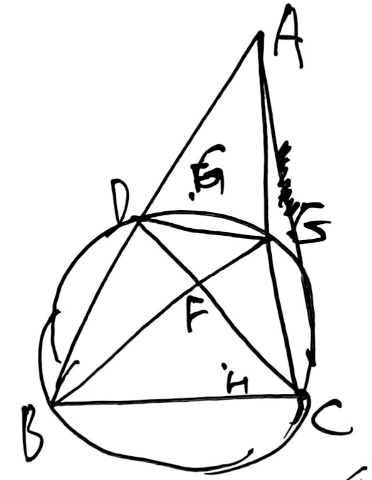
$\frac{c}{\sin C} = \frac{b}{\sin B}$

$BD = CD$

$\therefore \boxed{\frac{\sin C}{\sin\alpha} = \frac{\sin B}{\sin\beta}}$

牛顿定理

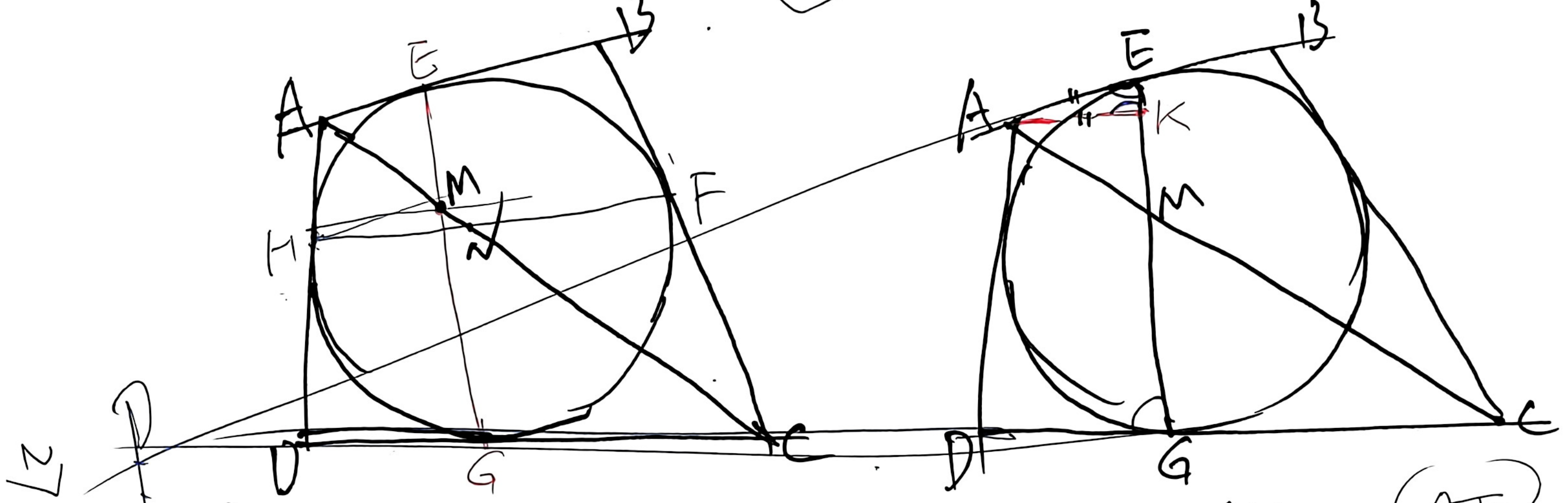




G.H为△ADB.△ABC垂心

求证 G.F.H 共线

遇比例作平行



分离：

刻画：$\frac{AM}{CM}=\frac{AN}{CN}$

执行：

$\frac{AM}{MC}=\frac{AK}{CG}=\frac{AE}{CG}$

同理 $\frac{AN}{CN}=\frac{AH}{CF}$

$\therefore \frac{AM}{MC}=\frac{AN}{CN}$ $\therefore$ M.N重合

$\therefore$ EG. FH. AC 共点

同理 EG. FH BD 共点

$\therefore$ ~~EG. FH. AC. BD~~ 共点

$$\frac{BI}{CL}=\frac{BE}{CD} \qquad \frac{EK}{DJ}=\frac{BE}{CD}$$

$\triangle IBE \sim \triangle LCD$ $\qquad$ $\triangle BEK \sim \triangle CDJ$

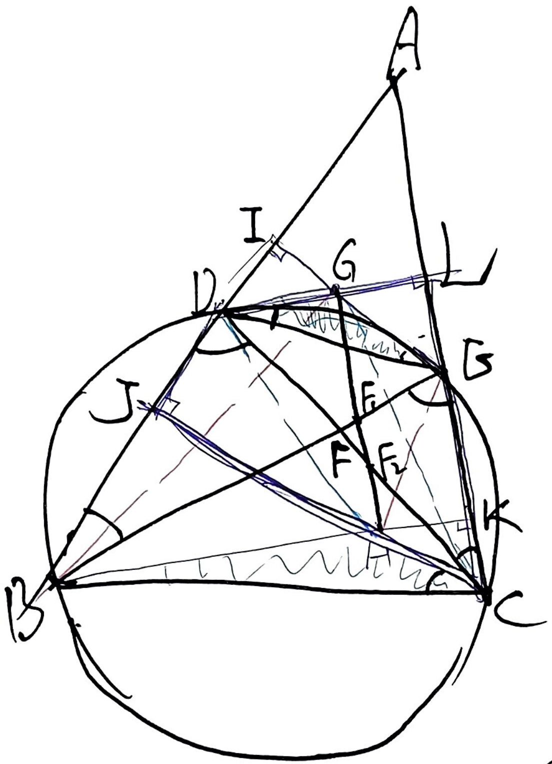
只需证 $\dfrac{EG}{BH}=\dfrac{DG}{CH}$

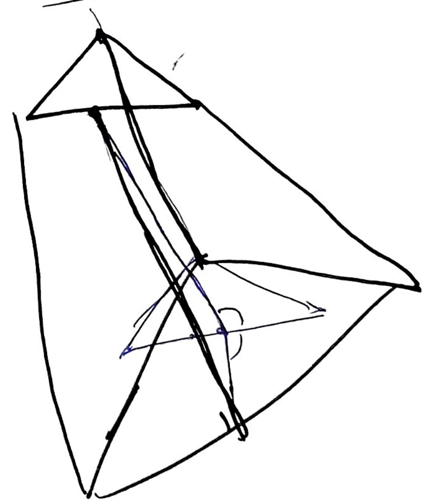
$\triangle ADE \sim \triangle ACB$

G. H 为对应点

D. C 对应点. $\qquad$ E. B 对应点

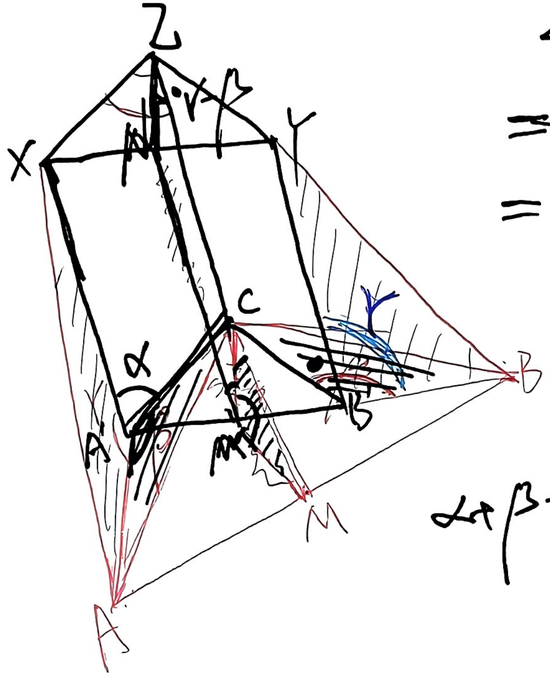
[illegible]





$$\frac{GF_1}{HF_1}=\frac{GF_2}{HF_2}\Leftrightarrow\frac{S_{\triangle GBE}}{S_{\triangle HBE}}=\frac{S_{\triangle GDC}}{S_{\triangle HDC}}$$

$$\Leftrightarrow\frac{BI\cdot EG}{BH\cdot EK}=\frac{DG\cdot CL}{CH\cdot DJ}$$



$\angle NM'C + \angle CM'M = \angle NZC + \angle CA'A$

$= \cancel{\angle XAC - \angle XZN + \angle CA'A}$

$= \cancel{\alpha - \angle XZN + \beta}$

$= \dfrac{\alpha+\beta-\gamma}{2} + \beta$

$= \dfrac{\alpha+\beta+\gamma}{2} = 180$

$\alpha+\beta+\gamma = 360$

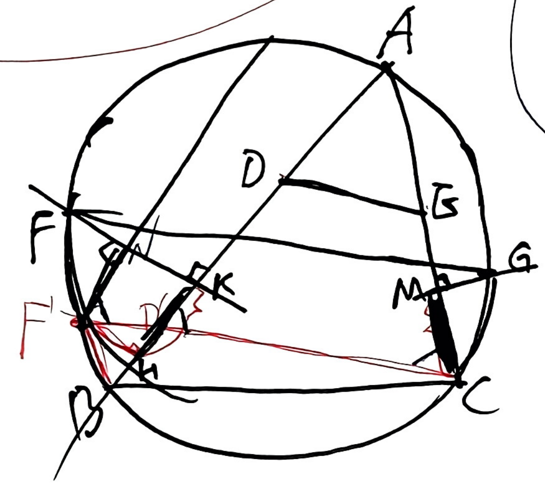
$\cancel{\angle XZN = \angle NZY = \gamma - \beta + \angle}$

$\angle NZC = \angle NZY - (\gamma - \beta)$

$= \angle XZC - \angle XZN = \alpha - \angle XZN$

$\therefore \angle NZC = \dfrac{\alpha - (\gamma - \beta)}{2} = \dfrac{\alpha+\beta-\gamma}{2}$

光速解法

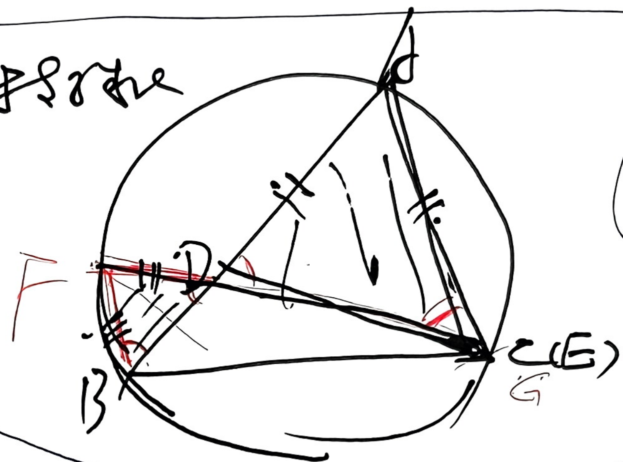


$F'C \parallel DE$

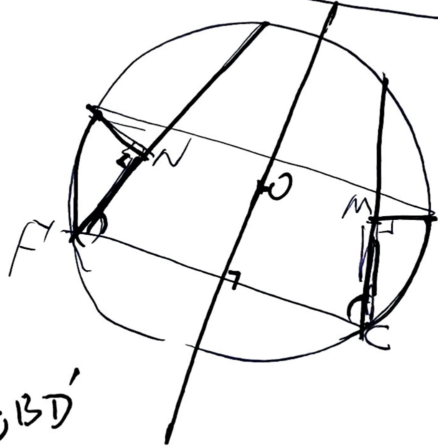
只需证 $F'C \parallel FG$

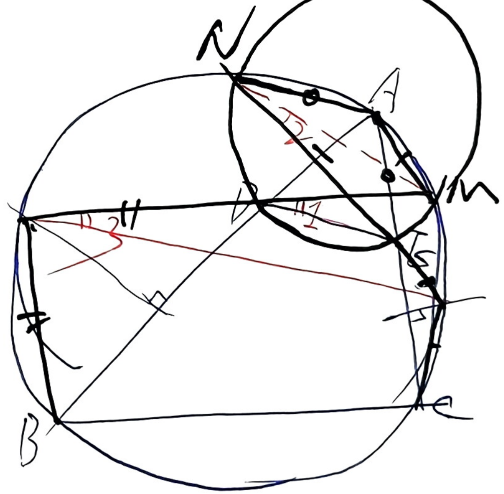
$HK = BK - BH = \frac{1}{2}BD - \frac{1}{2}BD'$

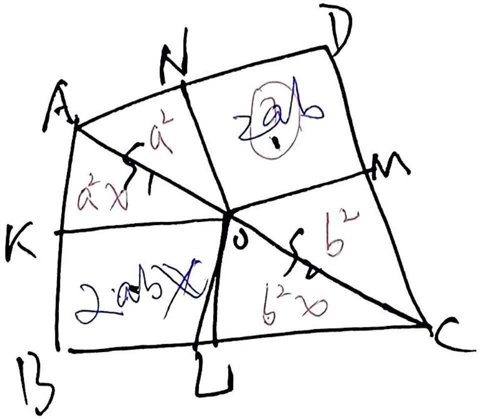
$= \frac{1}{2}DD' = \frac{1}{2}CE = CM$



(7)







$$\sqrt{S}=\sqrt{S_1}+\sqrt{S_2}$$

$$\frac{S_{\triangle AON}}{S_{\triangle ACD}}=\frac{S_{\triangle AOK}}{S_{\triangle ABC}}=\left(\frac{AO}{AC}\right)^2$$

$$\frac{S_{\triangle AON}+S_{\triangle AOK}}{S_{\triangle ACD}+S_{\triangle ABC}}=\left(\frac{AO}{AC}\right)^2$$

$$\frac{S_1}{S}=\left(\frac{AO}{AC}\right)^2 \Longrightarrow \frac{\sqrt{S_1}}{\sqrt{S}}=\frac{AO}{AC}$$

同理 $\frac{\sqrt{S_2}}{\sqrt{S}}=\frac{CO}{AC}$

$$\Longrightarrow \frac{\sqrt{S_1}+\sqrt{S_2}}{\sqrt{S}}=1$$

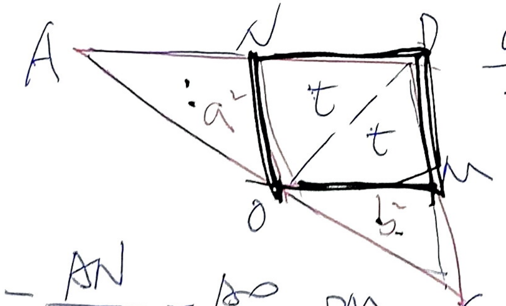
$$\Longrightarrow \sqrt{S_1}+\sqrt{S_2}=\sqrt{S}$$

反之: $\sqrt{S}=\sqrt{S_1}+\sqrt{S_2}$

$$\Longleftrightarrow S=S_1+S_2+2\sqrt{S_1S_2}$$

$$2ab+2abx=2\sqrt{a^2(1+x)\,b^2(1+x)}$$

$$2ab(1+x)=2ab(1+x)$$



$$\frac{a^2}{t}=\frac{t}{b^2}$$

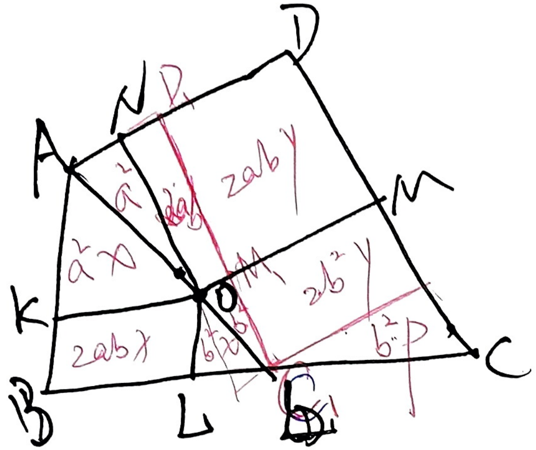
$$\Downarrow$$

$$t^2=a^2b^2$$

$$\frac{a^2}{t}=\frac{AN}{DN}=\frac{AO}{OC}=\frac{DM}{CM}=\frac{t}{b^2}$$

$$t=ab$$

$$S^2_{\square OMDN}=4S_{\triangle OAN}S_{\triangle OCM}$$



$$\sqrt{S} \geq \sqrt{S_1} + \sqrt{S_2}$$

$$\iff S \geq S_1 + S_2 + 2\sqrt{S_1 S_2}$$

$$\iff \underline{S - S_1 - S_2} \geq 2\sqrt{S_1 S_2}$$

$$\iff 2abx + 2ab + 2aby \geq 2\sqrt{a^2(1+x)\, b^2(1+x+2y+p)}$$

$$\iff 1+x+y \geq \sqrt{(1+x)(1+x+2y+p)}$$

$$\iff (1+x+y)^2 \geq (1+x)(1+x+2y+p)$$

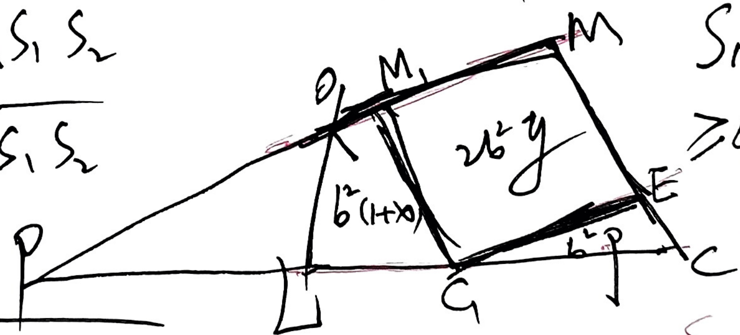
$$\iff (1+x+y)^2 \geq (1+x+y-y)(1+x+y+y+p)$$

$$\iff \cancel{(1+x+y)^2} \geq \cancel{(1+x+y)^2} - y(1+x+y) + (y+p)(1+x+y) - y(y+p)$$

$$\iff y(y+p) \geq p(1+x+y)$$

$$\iff b^4 y^2 \geq b^2 p\, b^2(1+x)$$

$$\iff (2b^2y)^2 \geq 4 b^2 p \cdot b^2(1+x)$$



$$S_{MM_1GE}^2 \geq 4 S_{\triangle CGE} \cdot S_{OM_1GL}$$

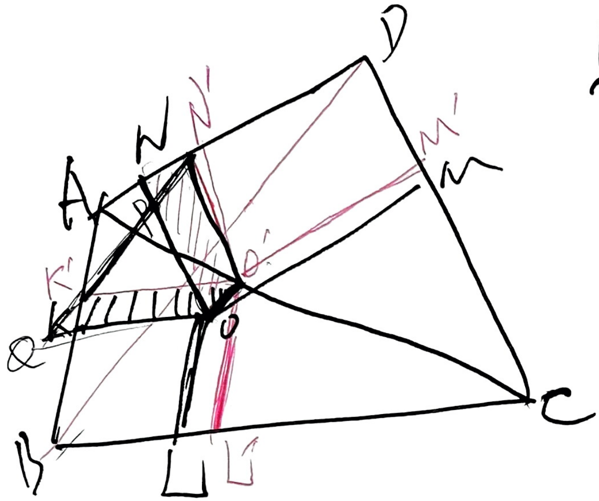
若$OM_1 \parallel CL$ 则等号成立

$S_{\triangle CGE} = 0$

更强结论

$$S_{MM_1GE} = 4 S_{\triangle PM_1G} S_{\triangle CGE}$$

$$> 4 S_{OM_1GL} S_{\triangle CGE}$$



$$\sqrt{S} = \sqrt{S_{O'K'AN'}} + \sqrt{S_{CL'O'M'}}$$

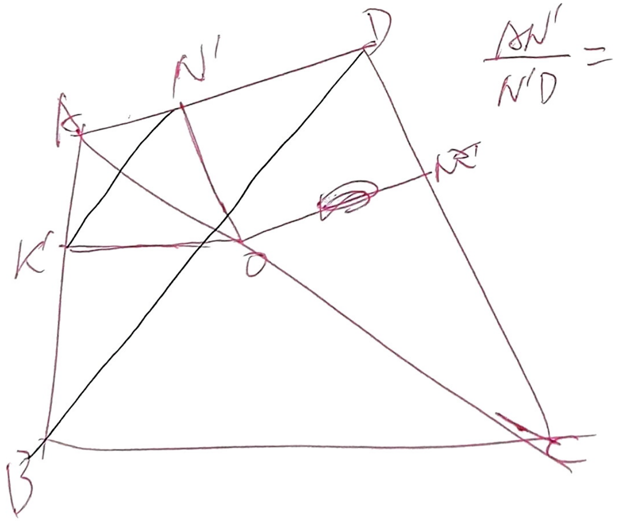
$$S_{O'K'AN'} \geq S_{OKAN}$$

$$\iff S_{OO'N'N} \geq S_{O'OKK'}$$

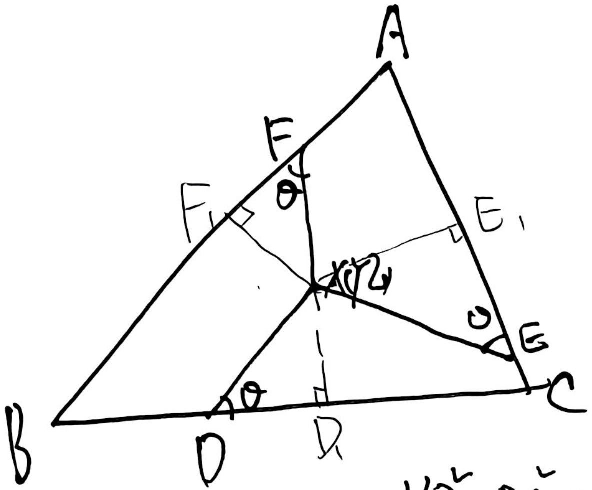
$$S_{CL'O'M'} \geq S_{CLOM}$$

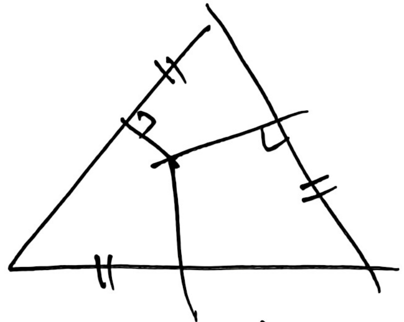
$$S_{OO'N'N} > S_{OO'N'P} = S_{\triangle OO'K'Q}$$

$$> S_{OO'K'K}$$



$$\frac{AN'}{N'D} = \frac{AO}{OC} = \frac{AK'}{BK'}$$





$$BD_1^2 - CD_1^2 + CE_1^2 - AE_1^2 + AF_1^2 - BF_1^2 = 0$$

$$\sqrt{S_{\triangle ABC}}\cos\theta + \frac{BD^2 - DC^2 + \cdots}{4\sqrt{S_{\triangle ABC}}} = 0$$

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$$\Leftrightarrow\ 4S_{\triangle ABC} = (CD^2 - BD^2 + EA^2 - CE^2 + FB^2 - AF^2)\tan\theta$$

右边 $= \left[(CD_1+DD_1)^2 - (BD_1-DD_1)^2 + (AE_1+EE_1)^2 - (CE_1-EE_1)^2 + (BF_1+FF_1)^2 - (AF_1-FF_1)^2\right]\tan\theta$

$= \left[\cancel{CD_1^2 - BD_1^2} + 2(CD_1+BD_1)DD_1 + \cancel{AE_1^2 - CE_1^2} + 2(AE_1+CE_1)EE_1 + \cancel{BF_1^2 - AF_1^2} + 2(BF_1+AF_1)FF_1\right]\tan\theta$

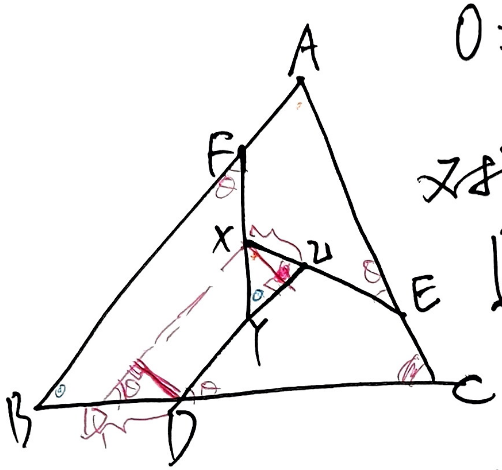
$= 2BC\cdot DD_1\tan\theta + 2AC\cdot EE_1\tan\theta + 2AB\cdot FF_1\tan\theta$

$= 2BC\cdot D_1X + 2AC\cdot E_1X + 2AB\cdot F_1X = 4S_{\triangle XBC} + 4S_{\triangle XAC} + 4S_{\triangle XAB}$

$\|$

$4S_{\triangle ABC}$

$$0=\sqrt{S_{\triangle ABC}}\cos\theta+\frac{BD_1^2-CD_1^2+CE^2-EA^2+AF^2-FB^2}{4\sqrt{S_{\triangle ABC}}}\sin\theta$$



对比

$$\sqrt{S_{\triangle XYZ}}=\sqrt{S_{\triangle ABC}}\cos\theta+\frac{BD^2-CD^2+CE^2-EA^2+AF^2-FB^2}{4\sqrt{S_{\triangle ABC}}}\sin\theta$$

只需证 $$\sqrt{S_{\triangle XYZ}}=\frac{BD^2-CD^2-(BD_1^2-CD_1^2)}{4\sqrt{S_{\triangle ABC}}}\sin\theta$$

$$4\sqrt{S_{\triangle XYZ}}\cdot\sqrt{S_{\triangle ABC}}=(BD^2-CD^2-BD_1^2+CD_1^2)\sin\theta$$

$\triangle XYZ\sim\triangle ABC$

$$\frac{\sqrt{S_{\triangle XYZ}}}{\sqrt{S_{\triangle ABC}}}=\frac{XZ}{AC}$$

$$\iff 4S_{\triangle ABC}\cdot\frac{XZ}{AC}=\left[(BD-CD)\cdot BC+(CD_1-BD_1)BC\right]\sin\theta$$

$$=BC(\underbrace{BD-BD_1}_{DD_1}+\underbrace{CD_1-CD}_{DD_1})\sin\theta$$

$$\iff 4S_{\triangle ABC}\frac{XZ}{AC}=2BC\cdot DD_1\sin\theta$$

$$\iff 4\cdot\frac{1}{2}\cancel{AC}\cdot\cancel{BC}\sin C\frac{XZ}{\cancel{AC}}=\cancel{2}\cancel{BC}\cdot DD_1\sin\theta$$

$$\iff XZ\sin C=DD_1\sin\theta$$